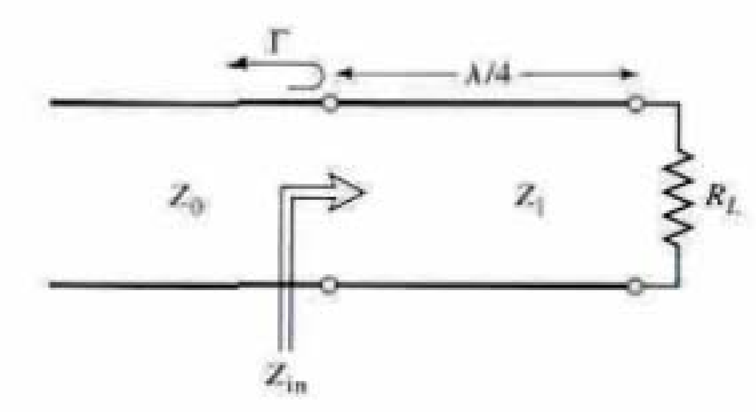
Adaptarea de Impedanta

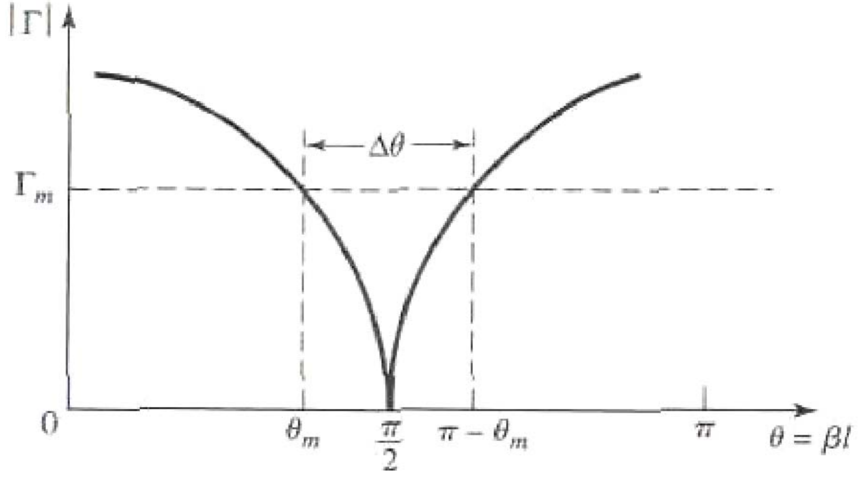
Transformatorul in sfert de lungime de unda

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$



$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_1(R_L - Z_0) + j\tan(\beta l)(Z_1^2 - Z_0 R_L)}{Z_1(R_L + Z_0) + j\tan(\beta l)(Z_1^2 + Z_0 R_L)}$$

$$Z_1 = \sqrt{Z_0 R_L}$$



$$|\Gamma| \approx \frac{|R_L - Z_0|}{2\sqrt{R_L Z_0}} |\cos\theta|$$

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 R_L}}{|R_L - Z_0|}$$

$$f_m = \frac{2\theta_m f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4}{\pi} \arccos\left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 R_L}}{|R_L - Z_0|}\right]$$

EXEMPLU

Să se proiecteze un transformator de adaptare cu o singură secțiune, care să adapteze o sarcină de 10Ω la o linie de 50Ω, la frecvența . Determinați banda fracțională pentru care . $SWR \leq 1.5$

$$Z_1 = \sqrt{Z_0 R_L} = \sqrt{50 \cdot 10} = 22.36\,\Omega$$

$$\Gamma_m = \frac{SWR - 1}{SWR + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2$$

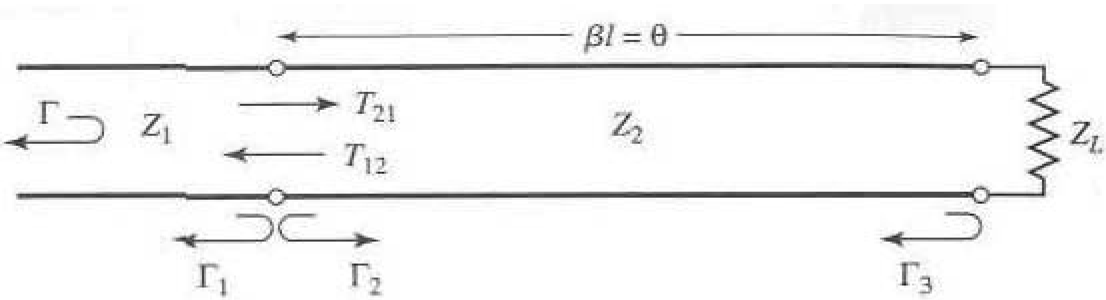
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \arccos\left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 R_L}}{\left| R_L - Z_0 \right|} \right] =$$

$$= 2 - \frac{4}{\pi} \arccos\left[\frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{\left| 10 - 50 \right|} \right] =$$

$$= 0.29$$

TEORIA REFLEXIILOR MICI

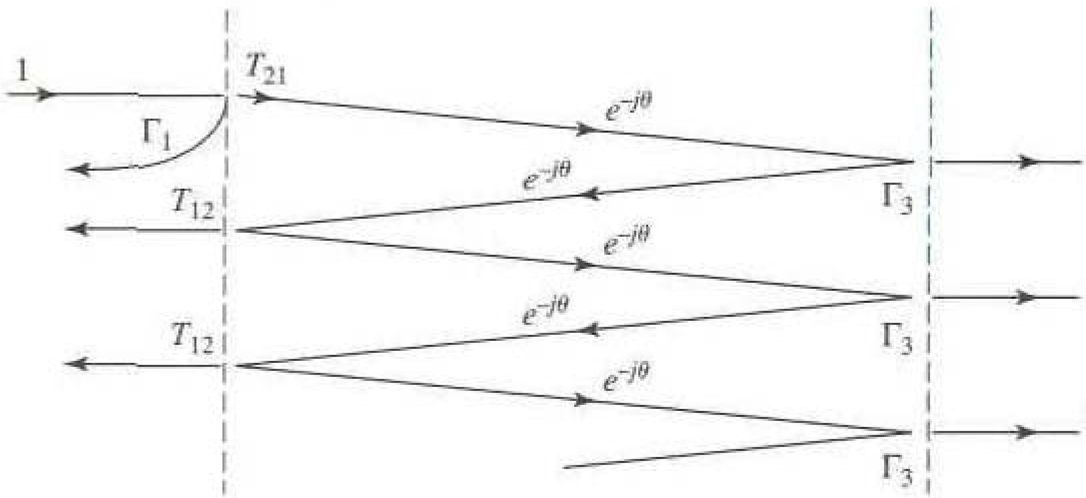
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad \Gamma_2 = -\Gamma_1 \qquad \Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2} \qquad T_{21} = 1 + \Gamma_1 = \frac{2Z_2}{Z_1 + Z_2} \qquad T_{12} = 1 + \Gamma_2 = \frac{2Z_1}{Z_1 + Z_2}$$



$$\Gamma = \Gamma_1 + T_{12}T_{21}\Gamma_3 e^{-2j\theta} + T_{12}T_{21}\Gamma_3^2\Gamma_2 e^{-4j\theta} + \ldots =$$

$$= \Gamma_1 + T_{12}T_{21}\Gamma_3 e^{-2j\theta}\sum_{n=0}^{\infty}\Gamma_2^n\Gamma_3^n e^{-2jn\theta}$$

$$\left|\Gamma_2\Gamma_3\right| << 1$$

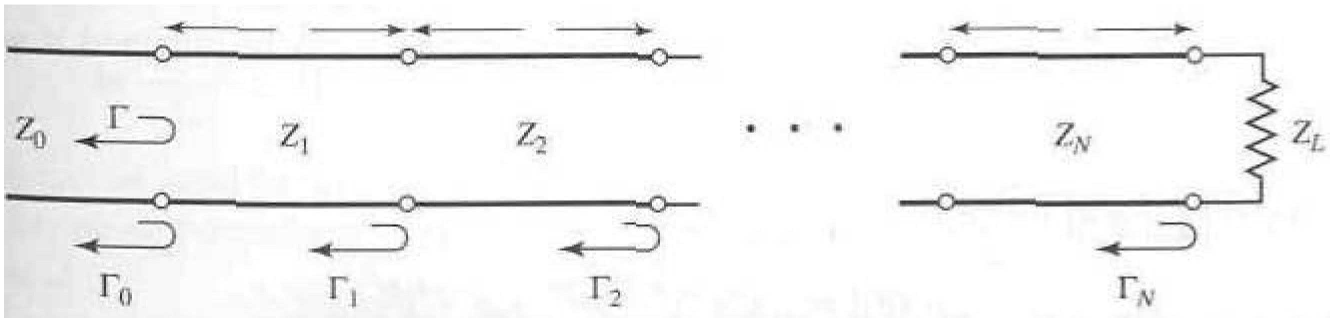


$$\Gamma = \Gamma_1 + \frac{T_{12}T_{21}\Gamma_3 e^{-2j\theta}}{1 - \Gamma_2\Gamma_3 e^{-2j\theta}}$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_3 e^{-2j\theta}}{1 + \Gamma_1\Gamma_3 e^{-2j\theta}}$$

$$\Gamma \approx \Gamma_1 + \Gamma_3 e^{-2j\theta}$$

Transformator cu mai multe secțiuni



$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \qquad \Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \qquad \Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 e^{-2j\theta} + \Gamma_2 e^{-4j\theta} + \cdots + \Gamma_N e^{-2jN\theta}$$

$$\Gamma(\theta) = 2e^{-jN\theta}\left[\Gamma_0 \cos(N\theta) + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_n \cos(N-2n)\theta + \cdots + \frac{1}{2}\Gamma_{N/2}\right]$$ N par

$$\Gamma(\theta) = 2e^{-jN\theta}\left[\Gamma_0 \cos(N\theta) + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_n \cos(N-2n)\theta + \cdots + \Gamma_{(N-1)/2} \cos\theta\right]$$ N impar

Transformatorul binomial

$$\Gamma(\theta) = A\left(1 + e^{-2j\theta}\right)^N \quad \Rightarrow \quad |\Gamma(\theta)| = |A|\left|e^{-j\theta}\right|^N\left|e^{j\theta} + e^{-j\theta}\right|^N = 2^N|A||\cos\theta|^N$$

$$\Gamma(0) = 2^N A = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \Rightarrow \quad \boxed{A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\Gamma(\theta) = A\left(1 + e^{-2j\theta}\right)^N = A\sum_{n=0}^{N} C_n^N e^{-2jn\theta}; \quad C_n^N = \frac{N!}{(N-n)!n!}$$

$$\Gamma(\theta) = A\sum_{n=0}^{N} C_n^N e^{-2jn\theta} = \Gamma_0 + \Gamma_1 e^{-2j\theta} + \Gamma_2 e^{-4j\theta} + \cdots + \Gamma_N e^{-2jN\theta}$$

$$\boxed{\Gamma_n = AC_n^N} \qquad \ln\frac{Z_{n+1}}{Z_n} \approx 2\Gamma_n = 2AC_n^N = 2\left(2^{-N}\right)\frac{Z_L - Z_0}{Z_L + Z_0}C_n^N \approx 2^{-N}C_n^N \ln\frac{Z_L}{Z_0}$$

$$\Gamma_m = 2^N|A|(\cos\theta_m)^N \quad \Rightarrow \quad \theta_m = \arccos\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right] \quad \Rightarrow \quad \frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{2f_m}{f_0} = 2 - \frac{4\theta_m}{\pi} =$$

$$= 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right]$$

EXEMPLU

- Să se proiecteze un transformator binomial cu trei secțiuni care să adapteze o sarcină de 50Ω la un fider de 100Ω şi să se calculeze banda de trecere pentru $\Gamma_m = 0.05$

Solutie

$$N = 3 \qquad Z_L = 50\Omega \qquad Z_0 = 100\Omega$$

$$A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}}\ln\frac{Z_L}{Z_0} = -0.0433$$

$$C_0^3 = \frac{3!}{3!0!} = 1 \qquad C_1^3 = \frac{3!}{2!1!} = 3 \qquad C_2^3 = \frac{3!}{1!2!} = 3$$

$n = 0$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_0^3 \ln\frac{Z_L}{Z_0} = \ln 100 + 2^{-3}(1)\ln\frac{50}{100} = 4.518$$

$Z_1 = 91.7\Omega$

$n = 1$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_1^3 \ln\frac{Z_L}{Z_0} = \ln 91.7 + 2^{-3}(3)\ln\frac{50}{100} = 4.26$$

$Z_2 = 70.7\Omega$

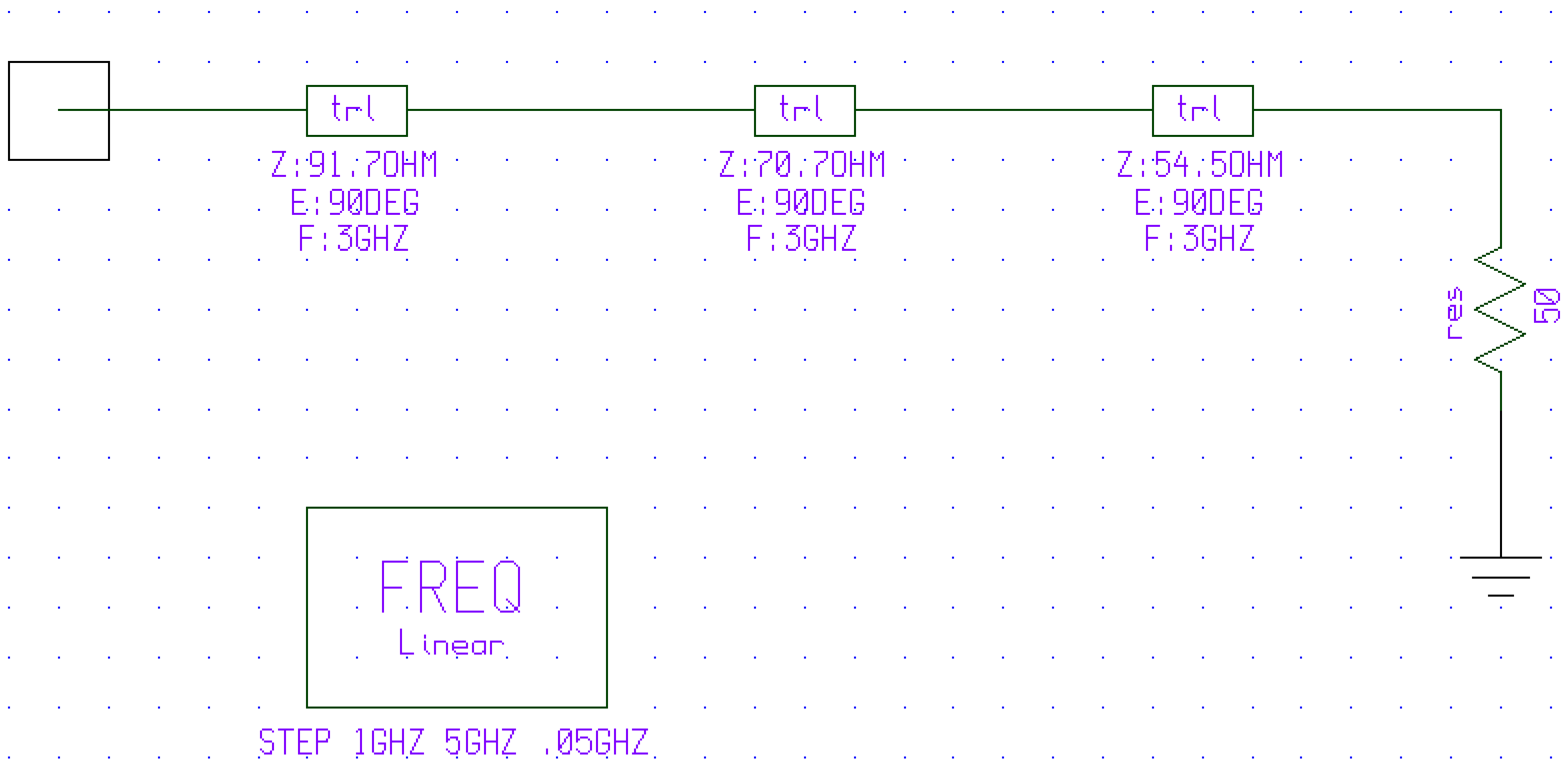
$n = 2$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_2^3 \ln\frac{Z_L}{Z_0} = \ln 70.7 + 2^{-3}(3)\ln\frac{50}{100} = 4.00$$

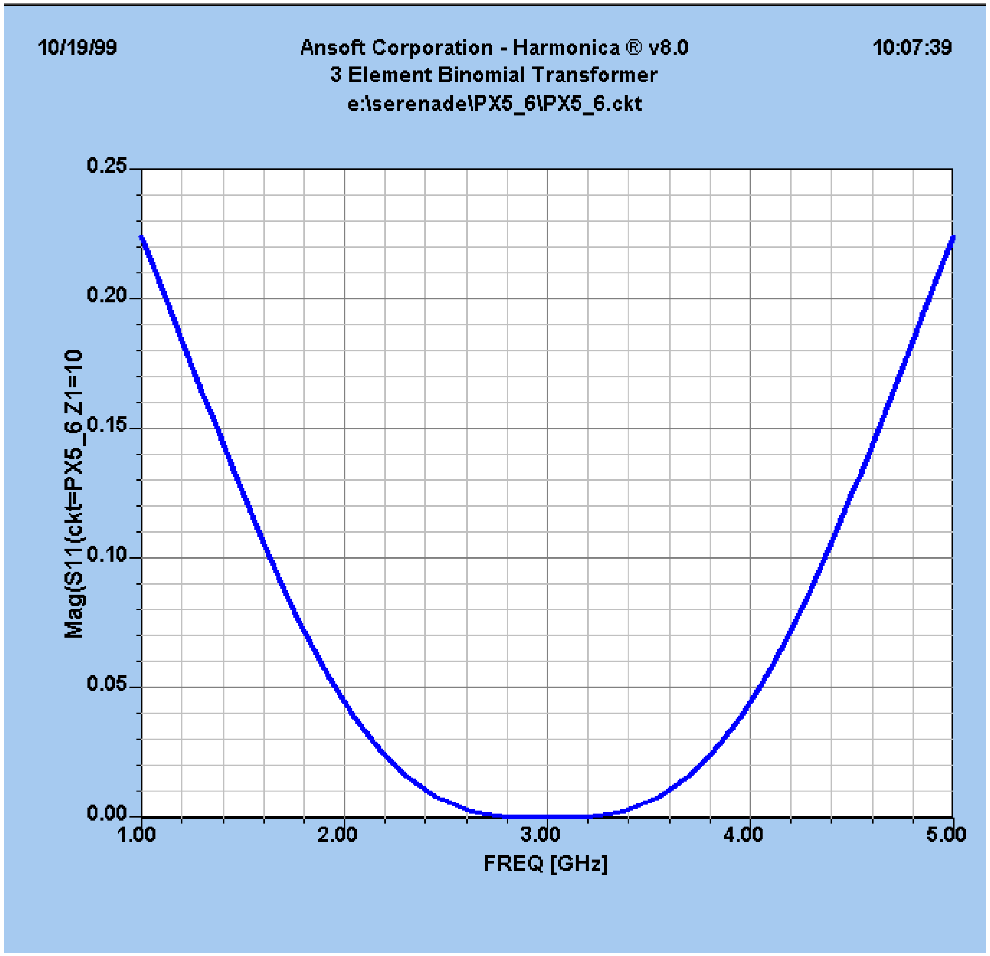
$Z_3 = 54.5\Omega$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right] = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{0.05}{0.0433}\right)^{1/3}\right] = 0.70$$

Circuitul


trl
Z:91.7OHM
E:90DEG
F:3GHZ
trl
Z:70.7OHM
E:90DEG
F:3GHZ
trl
Z:54.5OHM
E:90DEG
F:3GHZ
res
50
FREQ
Linear
STEP 1GHZ 5GHZ .05GHZ

Simularea


10/19/99
Ansoft Corporation - Harmonica ® v8.0
3 Element Binomial Transformer
e:\serenade\PX5_6\PX5_6.ckt
10:07:39
0.25
0.20
0.15
0.10
0.05
0.00
Mag[S11(ckt=PX5_6 Z1=10
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

Transformatorul Chebyshev

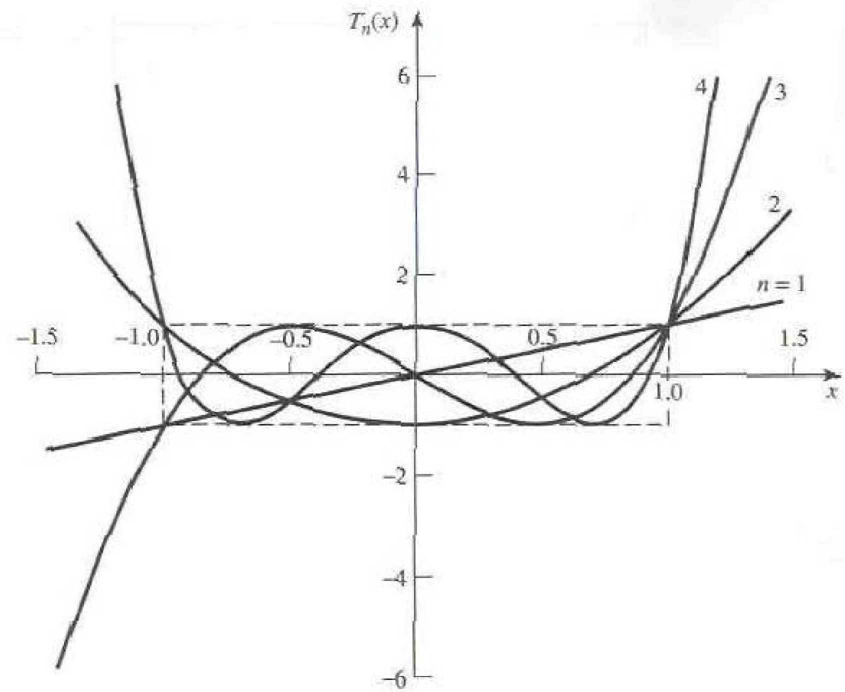
- Polomoame Cebyshev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$T_n(x) = \cosh(n \operatorname{arccos} h(x))$

Polinoamele Chebyshev folosite n proiectarea transformatorului de adaptare

$$x = \cos\theta$$

$$T_n(\cos\theta) = \cos(n\theta)$$

$$T_n(x) = \cos(n \arccos(x)) \qquad |x| < 1$$

$$T_n(x) = \cosh(n \operatorname{arccos} h(x)) \qquad |x| > 1$$

$$T_n\left(\frac{\cos\theta}{\cos\theta_m}\right) = T_n(\sec\theta_m \cos\theta) = \cos n\left[\arccos\left(\frac{\cos\theta}{\cos\theta_m}\right)\right]$$

$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m(1 + \cos 2\theta) - 1$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m(\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

$$T_4(\sec\theta_m \cos\theta) = \sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m(\cos 2\theta + 1) + 1$$

Proiectarea unui transformator de adaptare de tip Chebyshev

N = par

$$\Gamma(\theta) = 2e^{-jN\theta}\left[\Gamma_0 \cos(N\theta) + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_n \cos(N-2n)\theta + \cdots + \frac{1}{2}\Gamma_{N/2}\right] = Ae^{-jN\theta}T_N(\sec\theta_m \cos\theta)$$

N = impar

$$\Gamma(\theta) = 2e^{-jN\theta}\left[\Gamma_0 \cos(N\theta) + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_n \cos(N-2n)\theta + \cdots + \Gamma_{(N-1)/2} \cos\theta\right] = Ae^{-jN\theta}T_N(\sec\theta_m \cos\theta)$$

$$\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0} = AT_N(\sec\theta_m) \Rightarrow$$

$$A = \frac{Z_L - Z_0}{Z_L + Z_0}\frac{1}{T_N(\sec\theta_m)}$$

$$T_N(\sec\theta_m) = \frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| \cong \frac{1}{2\Gamma_m}\left|\ln\frac{Z_L}{Z_0}\right|$$

$$\Gamma_n \cong \frac{1}{2}\ln\frac{Z_{n+1}}{Z_n}$$

$$\sec\theta_m = \cosh\left[\frac{1}{N}\operatorname{arccos}h\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right] \cong \cosh\left[\frac{1}{N}\operatorname{arccos}h\left(\left|\frac{\ln(Z_L/Z_0)}{2\Gamma_m}\right|\right)\right] \Rightarrow \frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

Exemplu

- Să se proiecteze un transformator Chebyshev , cu trei sectiuni, care să adapteze o sarcină de 100 Ω la o linie de 50 Ω, cu un $\Gamma_m = 0.05$

Solutie

$$N = 3 \qquad Z_0 = 50\Omega \qquad Z_L = 100\Omega$$

$$\Gamma(\theta) = 2e^{-j3\theta}\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = Ae^{-j3\theta}T_3(\sec\theta_m \cos\theta)$$

$$A = \Gamma_m = 0.05$$

$$\sec\theta_m = \cosh\left[\frac{1}{N}\operatorname{arccos}h\left(\frac{\ln Z_L/Z_0}{2\Gamma_m}\right)\right] = \cosh\left[\frac{1}{3}\operatorname{arccos}h\left(\frac{\ln(100/50)}{2(0.05)}\right)\right] = 1.408 \qquad \theta_m = 44.7°$$

$$2\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = A\sec^3\theta_m(\cos 3\theta + 3\cos\theta) - 3A\sec\theta_m \cos\theta$$

$$\cos 3\theta \qquad 2\Gamma_0 = A\sec^3\theta_m \qquad \Gamma_0 = 0.0698 = \Gamma_3$$

$$\cos\theta \qquad 2\Gamma_1 = 3A(\sec^3\theta_m - \sec\theta_m) \qquad \Gamma_1 = 0.1037 = \Gamma_2$$

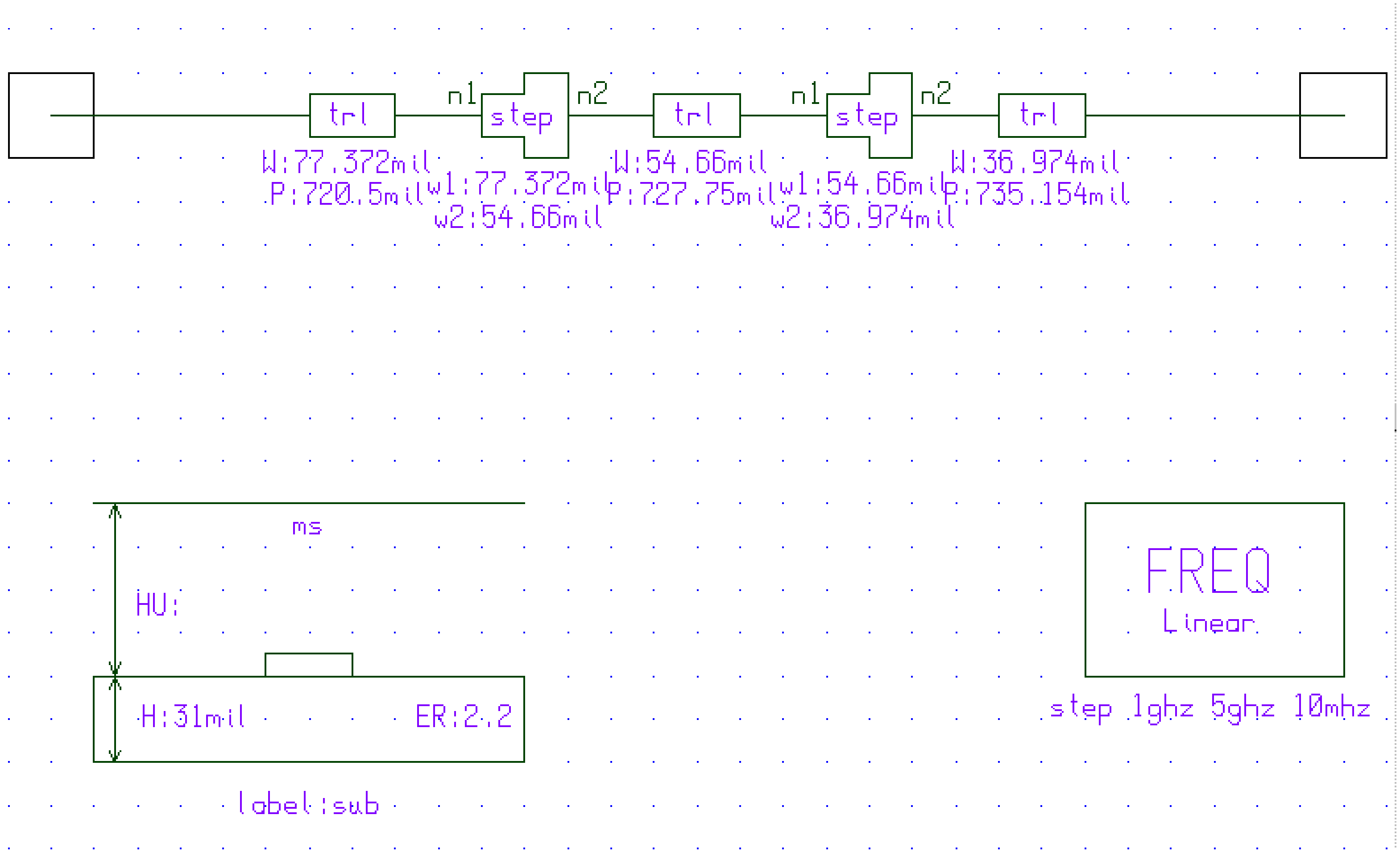
$$n = 0 \qquad \ln Z_1 = \ln Z_0 + 2\Gamma_0 = \ln 50 + 2(0.0698) = 4.051 \qquad Z_1 = 57.5\Omega$$

$$n = 1 \qquad \ln Z_2 = \ln Z_1 + 2\Gamma_1 = \ln 57.5 + 2(0.1037) = 4.259 \qquad Z_2 = 70.7\Omega$$

$$n = 2 \qquad \ln Z_3 = \ln Z_2 + 2\Gamma_2 = \ln 70.7 + 2(0.1037) = 4.466 \qquad Z_3 = 87\Omega$$

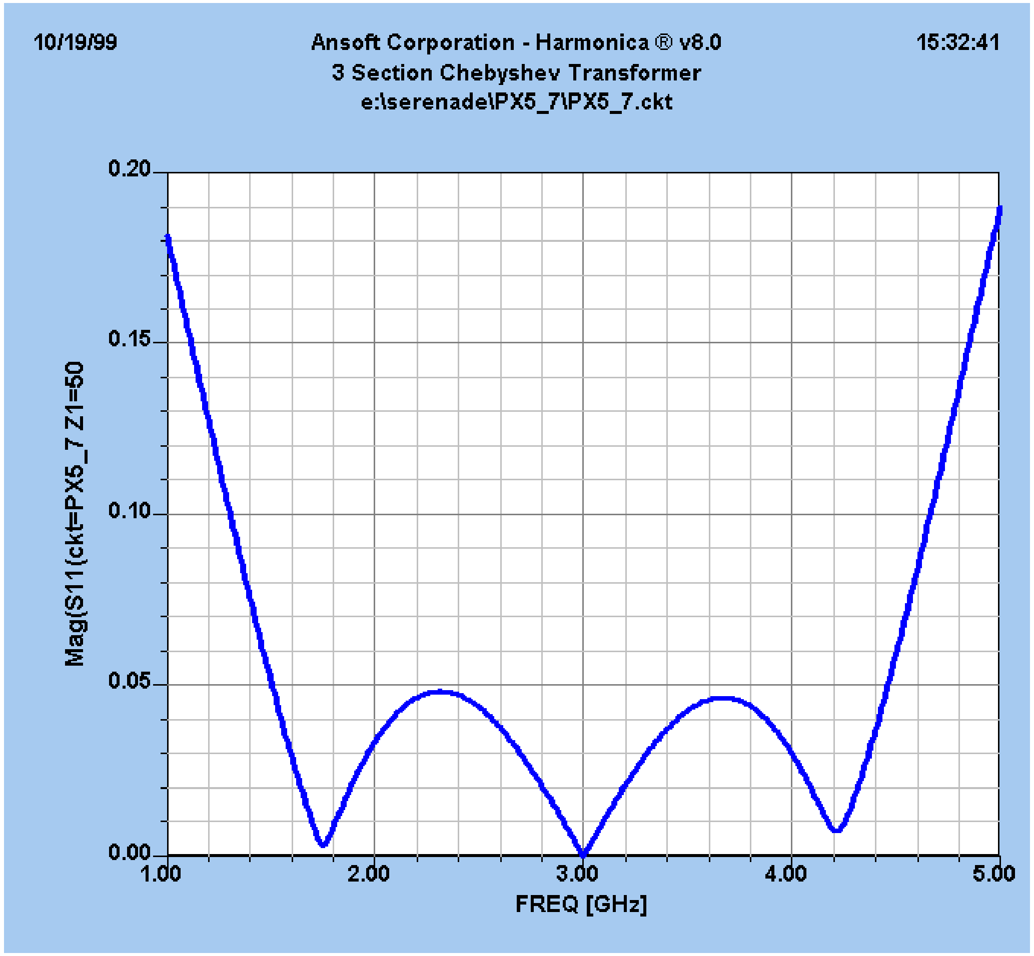
$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - 4\frac{44.7°}{180°} = 1.01$$

Circuitul



n1
n2
n1
n2
trl
step
trl
step
trl
W:77.372mil
P:720.5mil
w1:77.372mil
w2:54.66mil
W:54.66mil
P:727.75mil
w1:54.66mil
w2:36.974mil
W:36.974mil
P:735.154mil
ms
HU:
H:31mil
ER:2.2
label:sub
FREQ
Linear
step 1ghz 5ghz 10mhz

Simularea



10/19/99
Ansoft Corporation - Harmonica ® v8.0
3 Section Chebyshev Transformer
e:\serenade\PX5_7\PX5_7.ckt
15:32:41
0.20
0.15
0.10
0.05
0.00
1.00
2.00
3.00
4.00
5.00
Mag(S11(ckt=PX5_7 Z1=50
FREQ [GHz]