2- Adaptarea de Impedanta

EXEMPLU

- Să se proiecteze un transformator binomial cu trei secțiuni care să adapteze o sarcină de 50Ω la un fider de 100Ω şi să se calculeze banda de trecere pentru $\Gamma_m = 0.05$

Solutie

$$N = 3 \qquad Z_L = 50\Omega \qquad Z_0 = 100\Omega$$

$$A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}}\ln\frac{Z_L}{Z_0} = -0.0433$$

$$C_0^3 = \frac{3!}{3!0!} = 1 \qquad C_1^3 = \frac{3!}{2!1!} = 3 \qquad C_2^3 = \frac{3!}{1!2!} = 3$$

$n = 0$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_0^3 \ln\frac{Z_L}{Z_0} = \ln 100 + 2^{-3}(1)\ln\frac{50}{100} = 4.518$$

$Z_1 = 91.7\Omega$

$n = 1$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_1^3 \ln\frac{Z_L}{Z_0} = \ln 91.7 + 2^{-3}(3)\ln\frac{50}{100} = 4.26$$

$Z_2 = 70.7\Omega$

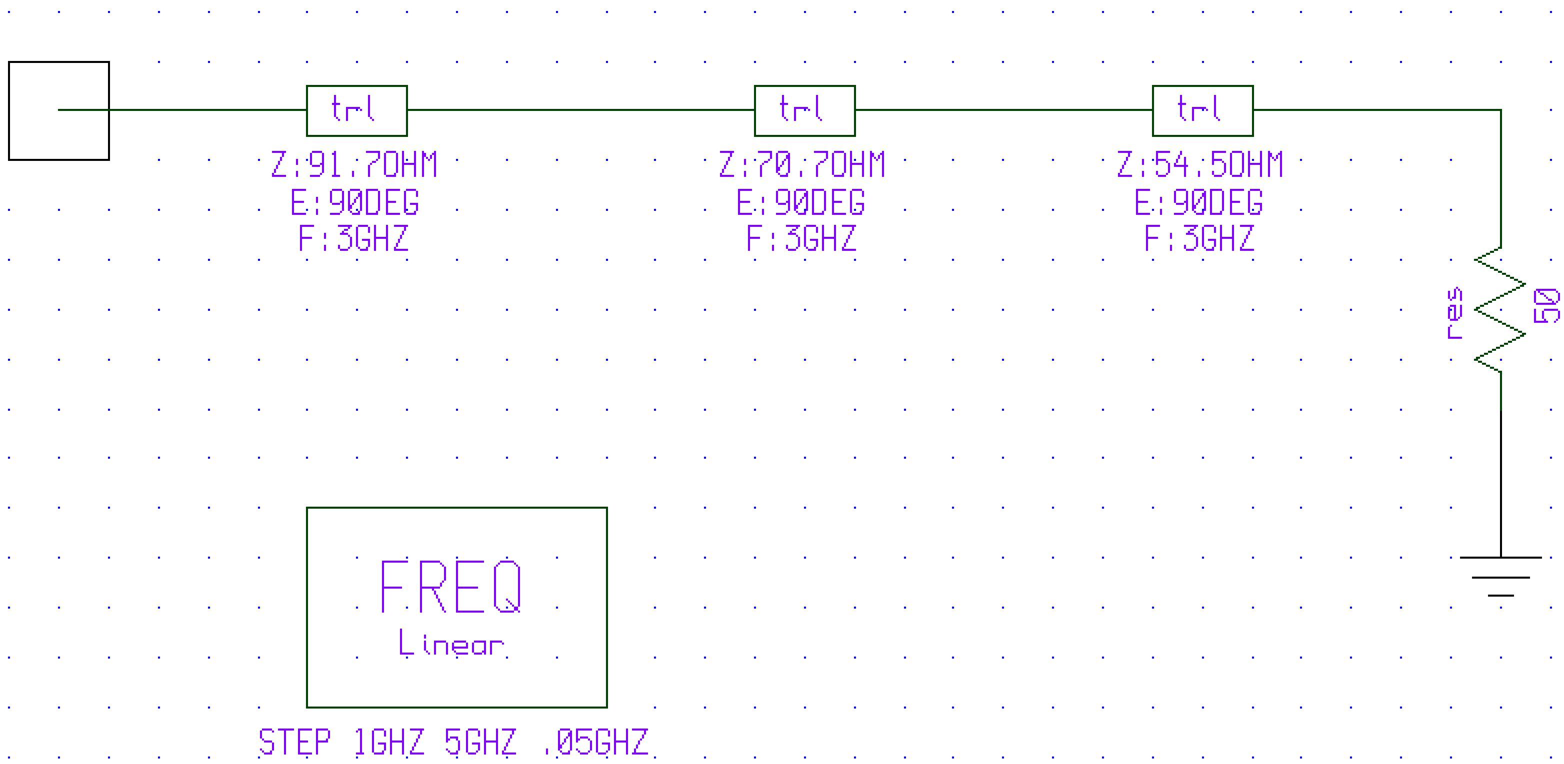
$n = 2$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_2^3 \ln\frac{Z_L}{Z_0} = \ln 70.7 + 2^{-3}(3)\ln\frac{50}{100} = 4.00$$

$Z_3 = 54.5\Omega$

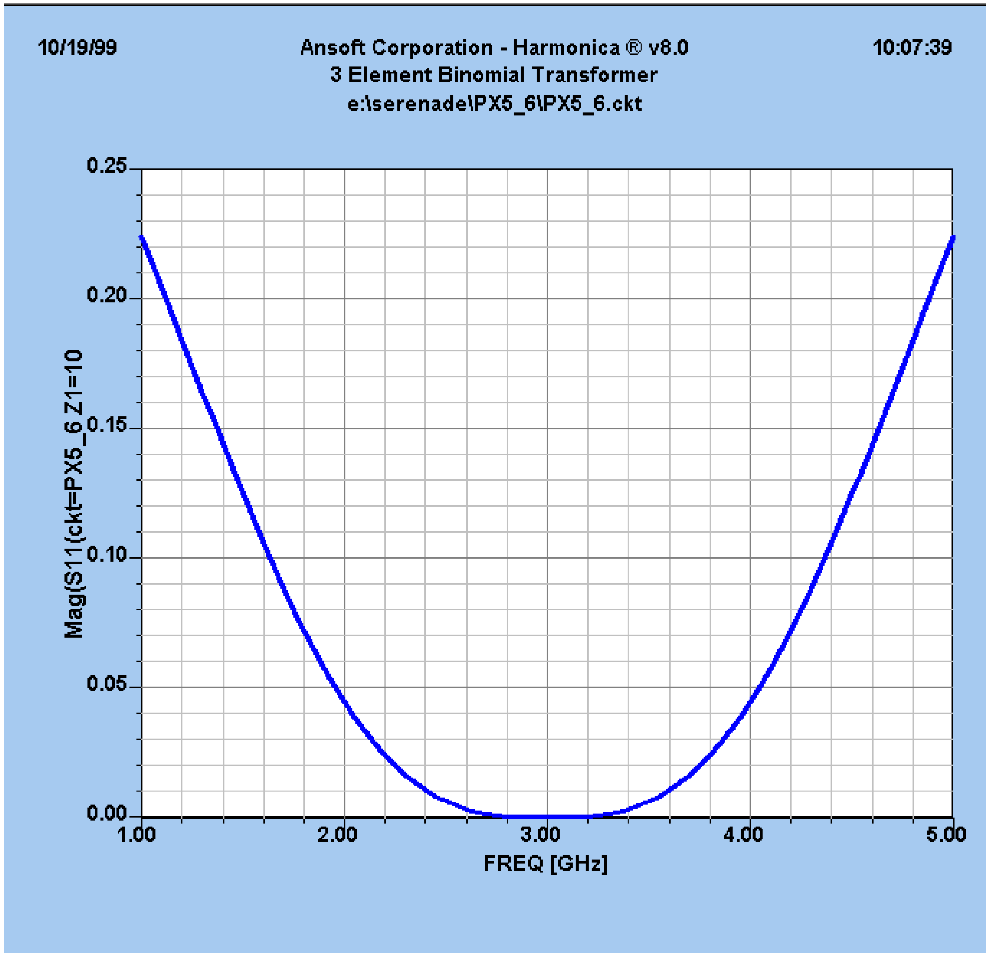
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right] = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{0.05}{0.0433}\right)^{1/3}\right] = 0.70$$

Circuitul



trl
Z:91.7OHM
E:90DEG
F:3GHZ
trl
Z:70.7OHM
E:90DEG
F:3GHZ
trl
Z:54.5OHM
E:90DEG
F:3GHZ
res
50
FREQ
Linear
STEP 1GHZ 5GHZ .05GHZ

Simularea


10/19/99
Ansoft Corporation - Harmonica ® v8.0
3 Element Binomial Transformer
e:\serenade\PX5_6\PX5_6.ckt
10:07:39
0.25
0.20
0.15
0.10
0.05
0.00
Mag[S11(ckt=PX5_6 Z1=10
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

Transformatorul Chebyshev

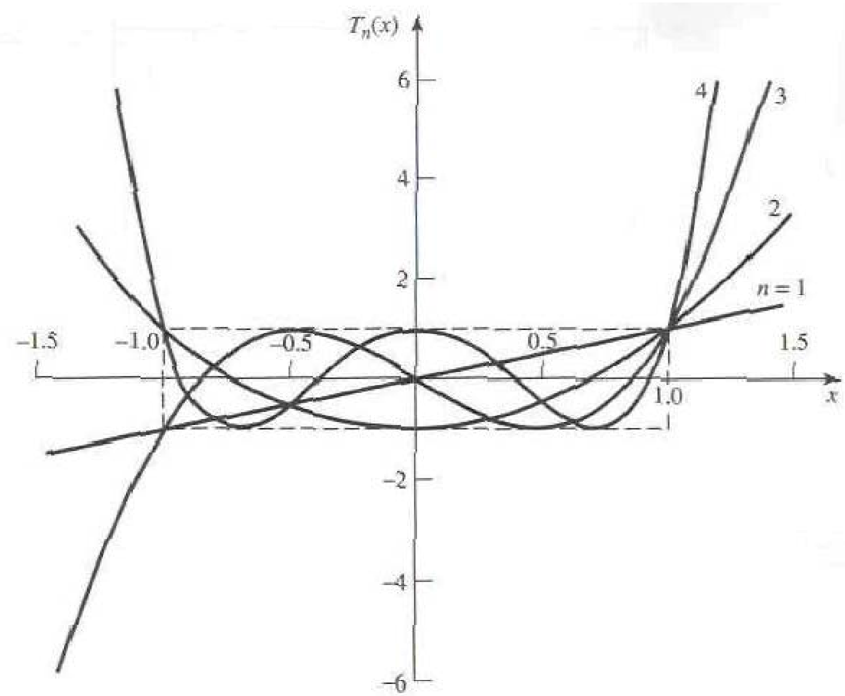
- Polomoame Cebyshev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$T_n(x) = \cosh(n \operatorname{arccos} h(x))$

Polinoamele Chebyshev folosite n proiectarea transformatorului de adaptare

$$x = \cos\theta$$

$$T_n(\cos\theta) = \cos(n\theta)$$

$$T_n(x) = \cos(n \arccos(x)) \qquad |x| < 1$$

$$T_n(x) = \cosh(n \operatorname{arccos} h(x)) \qquad |x| > 1$$

$$\boxed{T_n\left(\frac{\cos\theta}{\cos\theta_m}\right) = T_n(\sec\theta_m \cos\theta) = \cos n\left[\arccos\left(\frac{\cos\theta}{\cos\theta_m}\right)\right]}$$

$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m(1 + \cos 2\theta) - 1$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m(\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

$$T_4(\sec\theta_m \cos\theta) = \sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m(\cos 2\theta + 1) + 1$$

Proiectarea unui transformator de adaptare de tip Chebyshev

N = par

$$\Gamma(\theta)=2e^{-jN\theta}\left[\Gamma_0\cos(N\theta)+\Gamma_1\cos(N-2)\theta+\cdots+\Gamma_n\cos(N-2n)\theta+\cdots+\frac{1}{2}\Gamma_{N/2}\right]=Ae^{-jN\theta}T_N(\sec\theta_m\cos\theta)$$

N = impar

$$\Gamma(\theta)=2e^{-jN\theta}\left[\Gamma_0\cos(N\theta)+\Gamma_1\cos(N-2)\theta+\cdots+\Gamma_n\cos(N-2n)\theta+\cdots+\Gamma_{(N-1)/2}\cos\theta\right]=Ae^{-jN\theta}T_N(\sec\theta_m\cos\theta)$$

$$\Gamma(0)=\frac{Z_L-Z_0}{Z_L+Z_0}=AT_N(\sec\theta_m)\quad\Rightarrow$$

$$A=\frac{Z_L-Z_0}{Z_L+Z_0}\frac{1}{T_N(\sec\theta_m)}$$

$$T_N(\sec\theta_m)=\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\cong\frac{1}{2\Gamma_m}\left|\ln\frac{Z_L}{Z_0}\right|$$

$$\Gamma_n\cong\frac{1}{2}\ln\frac{Z_{n+1}}{Z_n}$$

$$\sec\theta_m=\cosh\left[\frac{1}{N}\operatorname{arccos}h\left(\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\right)\right]\cong\cosh\left[\frac{1}{N}\operatorname{arccos}h\left(\left|\frac{\ln(Z_L/Z_0)}{2\Gamma_m}\right|\right)\right]\quad\Rightarrow\quad\frac{\Delta f}{f_0}=2-\frac{4\theta_m}{\pi}$$

Exemplu

- Să se proiecteze un transformator Chebyshev , cu trei sectiuni, care să adapteze o sarcină de 100 Ω la o linie de 50 Ω, cu un $\Gamma_m = 0.05$

Solutie

$$N = 3 \qquad Z_0 = 50\Omega \qquad Z_L = 100\Omega$$

$$\Gamma(\theta) = 2e^{-j3\theta}\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = Ae^{-j3\theta}T_3\left(\sec\theta_m \cos\theta\right)$$

$$A = \Gamma_m = 0.05$$

$$\sec\theta_m = \cosh\left[\frac{1}{N}\operatorname{arccos}h\left(\frac{\ln Z_L/Z_0}{2\Gamma_m}\right)\right] = \cosh\left[\frac{1}{3}\operatorname{arccos}h\left(\frac{\ln(100/50)}{2(0.05)}\right)\right] = 1.408 \qquad \theta_m = 44.7°$$

$$2\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = A\sec^3\theta_m\left(\cos 3\theta + 3\cos\theta\right) - 3A\sec\theta_m \cos\theta$$

$$\cos 3\theta \qquad 2\Gamma_0 = A\sec^3\theta_m \qquad \Gamma_0 = 0.0698 = \Gamma_3$$

$$\cos\theta \qquad 2\Gamma_1 = 3A\left(\sec^3\theta_m - \sec\theta_m\right) \qquad \Gamma_1 = 0.1037 = \Gamma_2$$

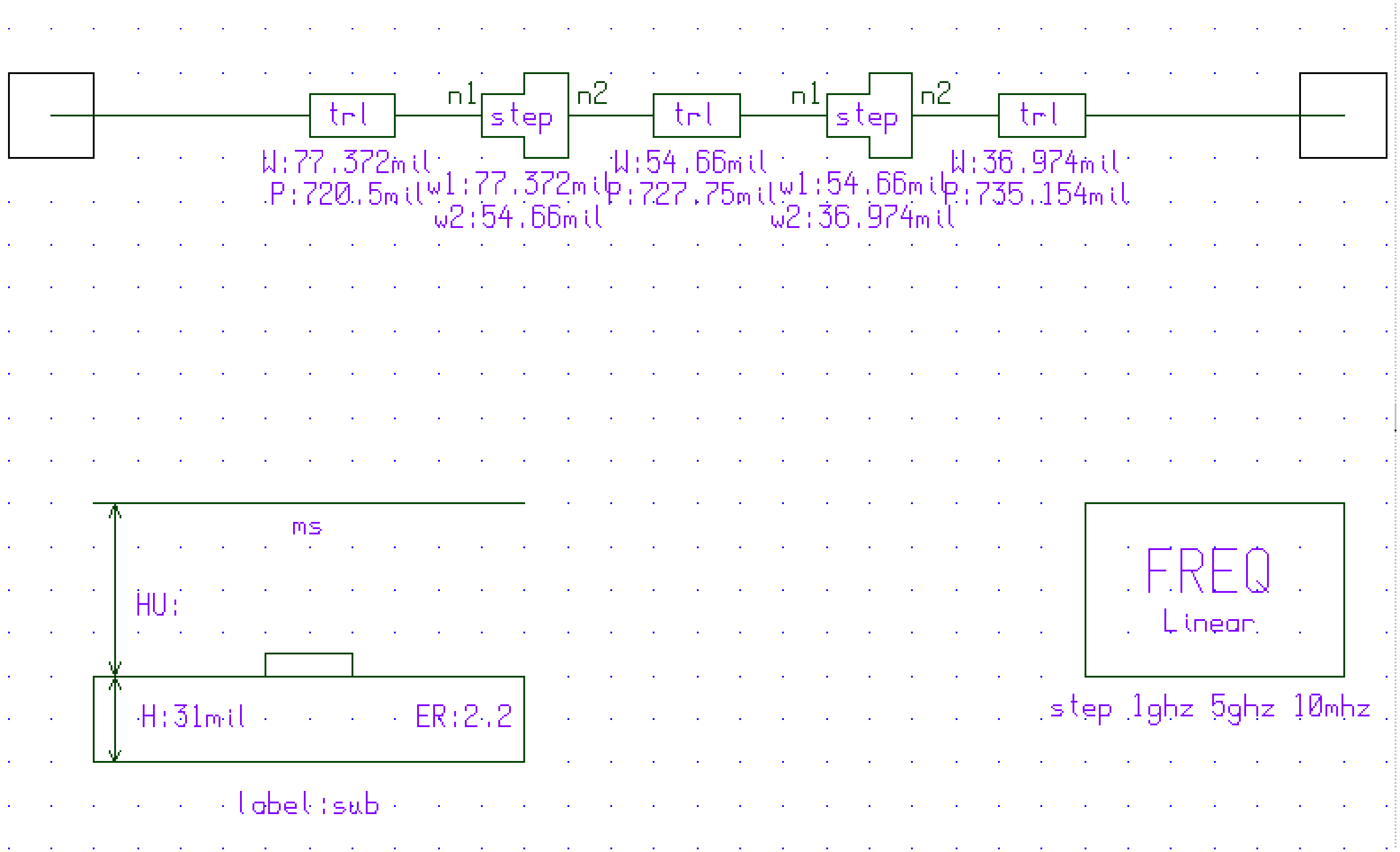
$$n = 0 \qquad \ln Z_1 = \ln Z_0 + 2\Gamma_0 = \ln 50 + 2(0.0698) = 4.051 \qquad Z_1 = 57.5\Omega$$

$$n = 1 \qquad \ln Z_2 = \ln Z_1 + 2\Gamma_1 = \ln 57.5 + 2(0.1037) = 4.259 \qquad Z_2 = 70.7\Omega$$

$$n = 2 \qquad \ln Z_3 = \ln Z_2 + 2\Gamma_2 = \ln 70.7 + 2(0.1037) = 4.466 \qquad Z_3 = 87\Omega$$

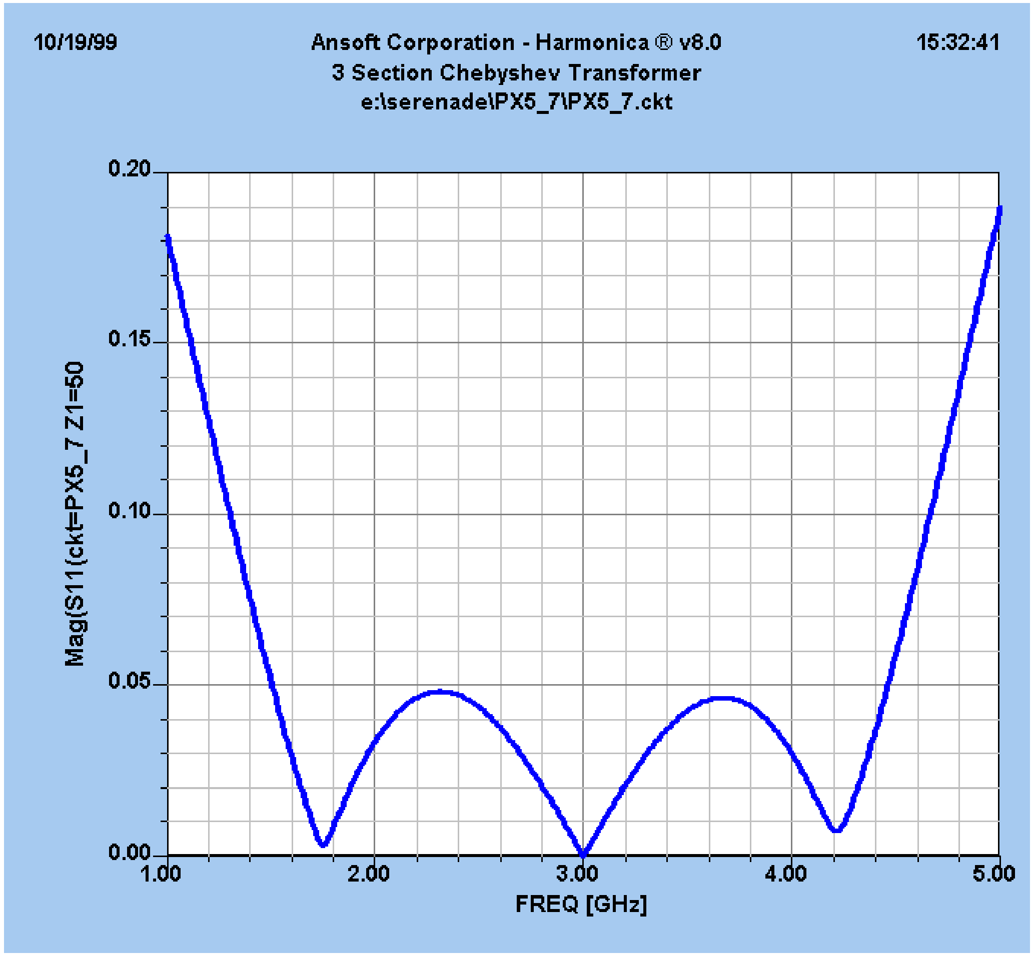
$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - 4\frac{44.7°}{180°} = 1.01$$

Circuitul



n1
n2
n1
n2
trl
step
trl
step
trl
W:77.372mil
P:720.5mil
w1:77.372mil
w2:54.66mil
W:54.66mil
P:727.75mil
w1:54.66mil
w2:36.974mil
W:36.974mil
P:735.154mil
ms
HU:
H:31mil
ER:2.2
label:sub
FREQ
Linear
step 1ghz 5ghz 10mhz

Simularea


10/19/99
Ansoft Corporation - Harmonica ® v8.0
3 Section Chebyshev Transformer
e:\serenade\PX5_7\PX5_7.ckt
15:32:41
0.20
0.15
0.10
0.05
0.00
Mag(S11(ckt=PX5_7 Z1=50
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

CUPLOARE

Proprietati de baza ale cuploarelor directionale

Circuite cu patru porti

$(S_{ij} = S_{ji})$ Reciproc

$S_{ii} = 0$ Adaptare simultana la toate portile

$>$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

+

$$\sum_k S_{ik} S_{kj}^* = \delta_{ij}$$ Fara pierderi

=

$<$ **10 ecuatii**

(11) $S_{14}^*\left(|S_{13}|^2 - |S_{24}|^2\right) = 0$

(13) $S_{23}\left(|S_{12}|^2 - |S_{34}|^2\right) = 0$

(14a) $|S_{12}|^2 + |S_{13}|^2 = 1$

(14b) $|S_{12}|^2 + |S_{24}|^2 = 1$

(14c) $|S_{13}|^2 + |S_{34}|^2 = 1$

(14d) $|S_{24}|^2 + |S_{34}|^2 = 1$

(15) $S_{12}^* S_{13} + S_{24}^* S_{34} = 0$

Cazul 1

(11) si (13) > $S_{14} = S_{23} = 0 \Rightarrow \quad [S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} \Leftrightarrow$ **Cuplor directional**

(14a) si (14b) > $|S_{13}| = |S_{24}|$

Alegem: $S_{12} = S_{34} = \alpha \quad S_{13} = \beta e^{j\theta} \quad S_{24} = \beta e^{j\phi}$

(14b) si (14d) > $|S_{12}| = |S_{34}|$

(15) > $\theta + \phi = \pi \pm 2n\pi$

Cuplor simetric $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

Cuplor antisimetric $\theta = 0\,, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

(14a) > $\alpha^2 + \beta^2 = 1$

Cazul 2

(11) si (13) > $\begin{cases} |S_{13}| = |S_{24}| \\ |S_{12}| = |S_{34}| \end{cases}$ Alegem: $S_{13} = S_{24} = \alpha \qquad S_{12} = S_{34} = j\beta$

(14a) > $\alpha^2 + \beta^2 = 1$

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \quad \Rightarrow \quad \alpha\left(S_{23} + S_{14}^*\right) = 0$$

$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \quad \Rightarrow \quad \beta\left(S_{14}^* - S_{23}\right) = 0$$

$\longrightarrow$ $S_{14} = S_{23} = 0$ **Cuplor directional**

$$[S] = \begin{bmatrix} 0 & j\beta & \alpha & 0 \\ j\beta & 0 & 0 & \alpha \\ \alpha & 0 & 0 & j\beta \\ 0 & \alpha & j\beta & 0 \end{bmatrix}$$

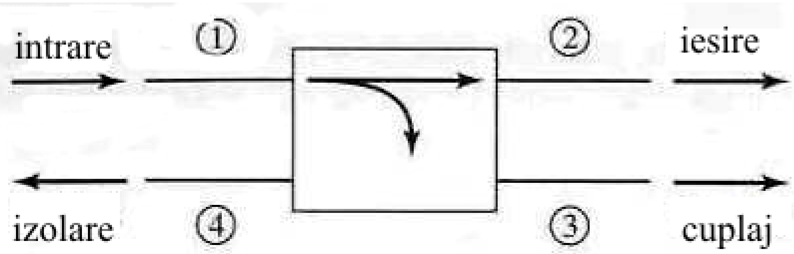
$\downarrow$

$\alpha = \beta = 0$ **Caz banal**

CONCLUZIE

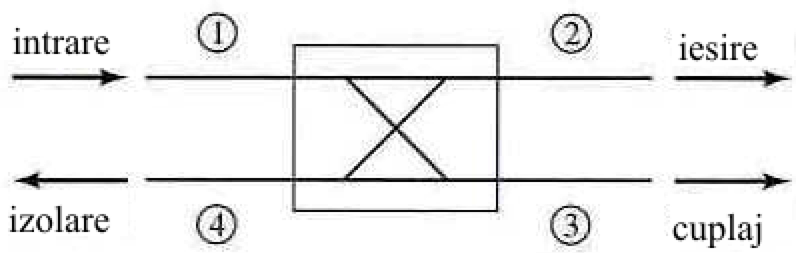
**Orice circuit cu patru porti,
reciproc, fara pierderi si adaptat la toate portile
este un cuplor directional**

Cuplor directional



$$|S_{12}|^2 = \alpha^2 = 1-\beta^2$$

$$|S_{13}|^2 = \beta^2$$



Cuplaj $= C = 10\log\dfrac{P_1}{P_3} = -20\log(\beta)\ dB$

Directivitate $= D = 10\log\dfrac{P_3}{P_4} = 20\log\left(\dfrac{\beta}{|S_{14}|}\right) dB$

Izolare $= I = 10\log\left(\dfrac{P_1}{P_4}\right) = -20\log|S_{14}|\ dB$

$$I = D + C\ ,\ \ dB$$

Cuplor hibrid

Cuplorul hibrid este cuplorul directional de 3 dB

$$\alpha = \beta = 1/\sqrt{2}$$

Cuplor hibrid in cuadratura

$$(\theta = \phi = \pi/2)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

Cuplor hibrid in inel

$$(\theta = 0\,, \phi = \pi)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$