EMITATOARE OPTICE

STARE TERMODINAMICA

$$f\left(p, V, T\right) = 0$$

Exemplu: Gazul ideal

$$pV - \frac{m}{M} RT = 0$$

$$R = 8.3\left[J/mol \cdot {}^{\circ}K\right]$$

$$N_A = 6.023 \cdot 10^{23}\left[mol^{-1}\right]$$

Principiul 1 al termodinamicii

$$\Delta U + L = Q$$

$$dU + dL = dQ$$

$$dU + pdV = dQ$$

$$\left(\frac{\partial U}{\partial T}\right)_V \neq \left(\frac{\partial U}{\partial T}\right)_p$$

Principiul 1 al termodinamicii
Exemplu

$$\left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + pdV = dQ \quad (1)$$

$$\left(\frac{\partial U}{\partial T}\right)_p dT + \left(\frac{\partial U}{\partial p}\right)_T dp + p\left(\frac{\partial V}{\partial T}\right)_p dT + p\left(\frac{\partial V}{\partial p}\right)_T dp = dQ \quad (2)$$

$$\left(\frac{\partial U}{\partial V}\right)_p dV + \left(\frac{\partial U}{\partial p}\right)_V dp + pdV = dQ \quad (3)$$

Capacitate termica = $\frac{dQ}{dT}$

Principiul 2 al termodinamicii

O transformare a carei singur rezultat final este sa transfere caldura de la un corp aflat la o anumita temperatura, la un corp aflat la o temperatura mai ridicata, este imposibila.

ENTROPIA

$$\begin{cases} \sum_{i=1}^{n} \frac{Q_i}{T_i} < 0 \\ \sum_{i=1}^{n} \frac{Q_i}{T_i} = 0 \end{cases}$$

Ciclu reversibil

$$\begin{cases} \oint \frac{dQ}{T} < 0 \\ \oint \frac{dQ}{T} = 0 \; (4) \end{cases}$$

Ciclu reversibil

$$(5) \int_A^B \frac{dQ}{T}$$

$$(6)\; S(A) = \int_O^A \frac{dQ}{T}$$

Entropia starii A

$$(7)\; S(B) - S(A) = \int_A^B \frac{dQ}{T}$$

Propietati Ale Entropiei

$$(8)\begin{cases} S(B)-S(A)=\int_A^B \frac{dQ}{T} & \text{Transformare reversivila} \\ S(B)-S(A)>\int_A^B \frac{dQ}{T} & \text{Transformare ireversivila} \end{cases}$$

$$(9)\begin{cases} dQ=0 \\ S(B)>S(A) \\ S(B)=S(A) \end{cases}$$

Sistem izolat

Transformare ireversibila intr-un sistem izolat

Transformare reversibila intr-un sistem izolat

Principiul 3 al termodinamicii

$$S = k\ln(W) + const. \quad (10)$$

$$k = \frac{R}{N_A} = 1.38\cdot 10^{-23}\,[J/°K]$$

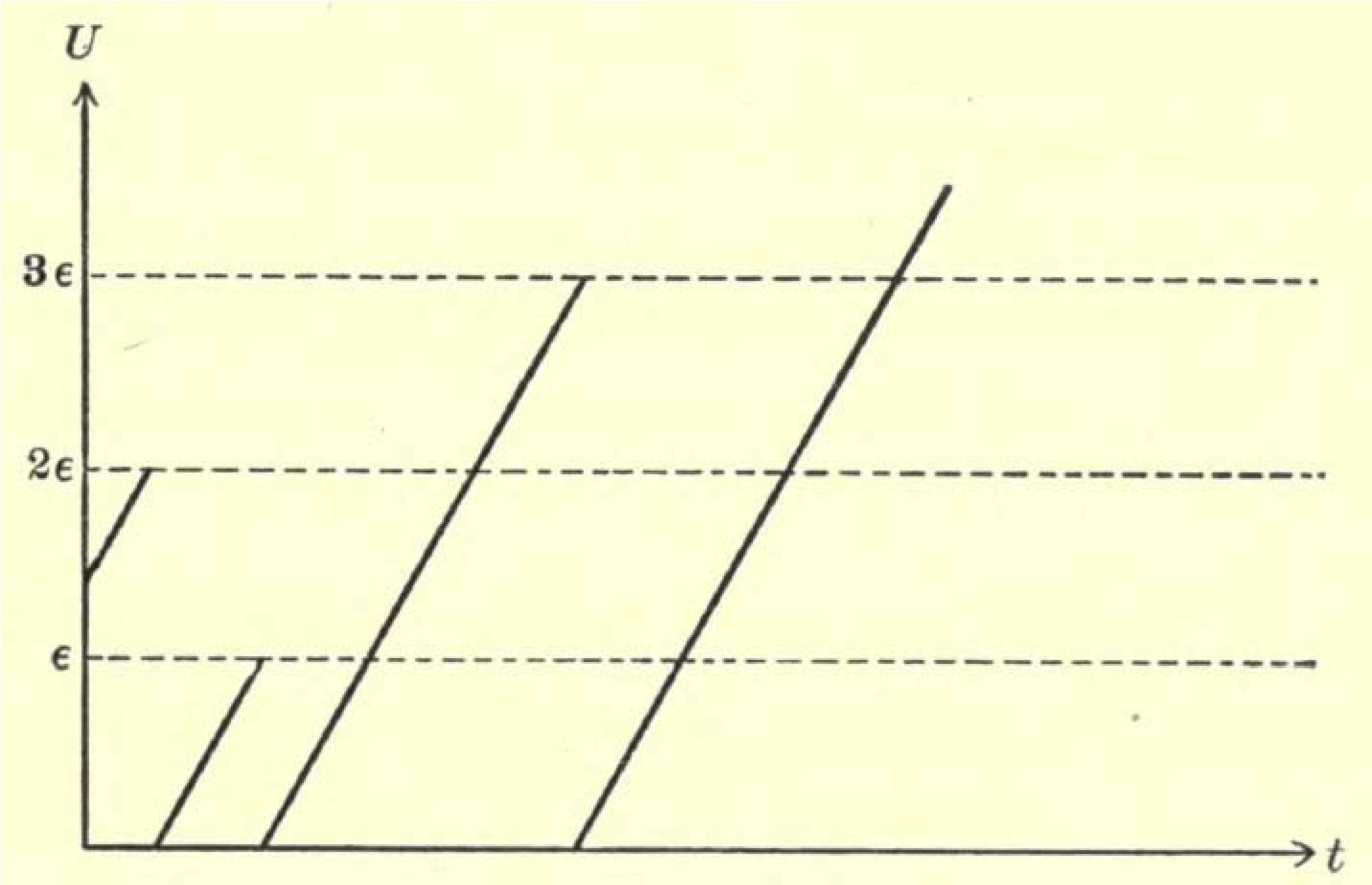
$$S(A) = \int_{O}^{A} \frac{dQ}{T}$$

$$(11)\; S(A) = \int_{T=0}^{A} \frac{dQ}{T}$$

$$(12)\; S(A) = k\ln W$$

Observatie: $S = 0 \Rightarrow W = 1$

Ipoteza cuantelor a lui Planck



$$U = n\,\varepsilon \qquad (13)$$

Determinarea probabilitati termodinamice

$$S(A) = k \log W \quad (14)$$

$$U_N = NU \quad (15)$$

$$S_N = NS \quad (16)$$

$$U_N = NU = Nn\varepsilon = P\varepsilon \quad (17)$$

$$S_N = k \log W \quad (18)$$

Calculul probabilitatii W

$$U_N = P\varepsilon$$

1	2	3	4	5	6	7	8	9	10
7	38	11	0	9	2	20	4	4	5

Tabel 1

$$R = \frac{N(N+1)(N+2)....(N+P-1)}{1 \cdot 2 \cdot 3 \cdots P} = \frac{(N+P-1)!}{(N-1)!P!} \quad (19)$$

$$N! \approx N^N \quad (20)$$

$$R \approx \frac{(N+P)^{N+P}}{N^N P^P} \quad (21)$$

Calculul probabilitatii W

$$S_N = k \ln W = k \ln R =$$

$$= k\left\{\left(N+P\right)\ln\left(N+P\right) - N\ln N - P\ln P\right\}$$

$$S_N = kN\left\{\left(1+\frac{P}{N}\right)\ln N\left(1+\frac{P}{N}\right) - \ln N - \frac{P}{N}\ln N\frac{P}{N}\right\} =$$

$$= kN\left\{\left(1+\frac{U}{\varepsilon}\right)\ln\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\ln\frac{U}{\varepsilon}\right\}$$

$$S = k\left\{\left(1+\frac{U}{\varepsilon}\right)\ln\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\ln\frac{U}{\varepsilon}\right\} \quad (22)$$

Legea de deplasare a lui Wien

$$u = \frac{\nu^3}{c^3} f\left(\frac{T}{\nu}\right) \quad (23)$$

$$u = \frac{8\pi\nu^2}{c^3} U \quad (24)$$

$$U = \nu f\left(\frac{T}{\nu}\right) \quad (25)$$

$$T = \nu f_1\left(\frac{U}{\nu}\right) \quad (26)$$

$$\frac{1}{T} = \frac{dS}{dU} \quad (27) \rightarrow \frac{dS}{dU} = \frac{1}{\nu} f_2\left(\frac{U}{\nu}\right) \quad (28) \rightarrow S = f_3\left(\frac{U}{\nu}\right) \quad (29)$$

Expresia quantei de radiatie

$$S = f\left(\frac{U}{\nu}\right) \quad (30)$$

$$S = k\left\{\left(1+\frac{U}{\varepsilon}\right)\ln\left(1+\frac{U}{\varepsilon}\right)-\frac{U}{\varepsilon}\ln\frac{U}{\varepsilon}\right\} \quad (31)$$

$$\varepsilon = h\nu$$

$$S = k\left\{\left(1+\frac{U}{h\nu}\right)\ln\left(1+\frac{U}{h\nu}\right)-\frac{U}{h\nu}\ln\frac{U}{h\nu}\right\} \quad (32)$$

Legea lui Plank

$$\frac{1}{T} = \frac{dS}{dU} \quad (33)$$

$$\frac{1}{T} = \frac{k}{h\nu} \ln\left(1 + \frac{h\nu}{U}\right) \quad (34)$$

$$U = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \quad (35)$$

Derivate ale legii lui Planck

$$U = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$h\nu \gg kT$$

$$\frac{1}{e^{\frac{h\nu}{kT}} - 1} \approx e^{-\frac{h\nu}{kT}}$$

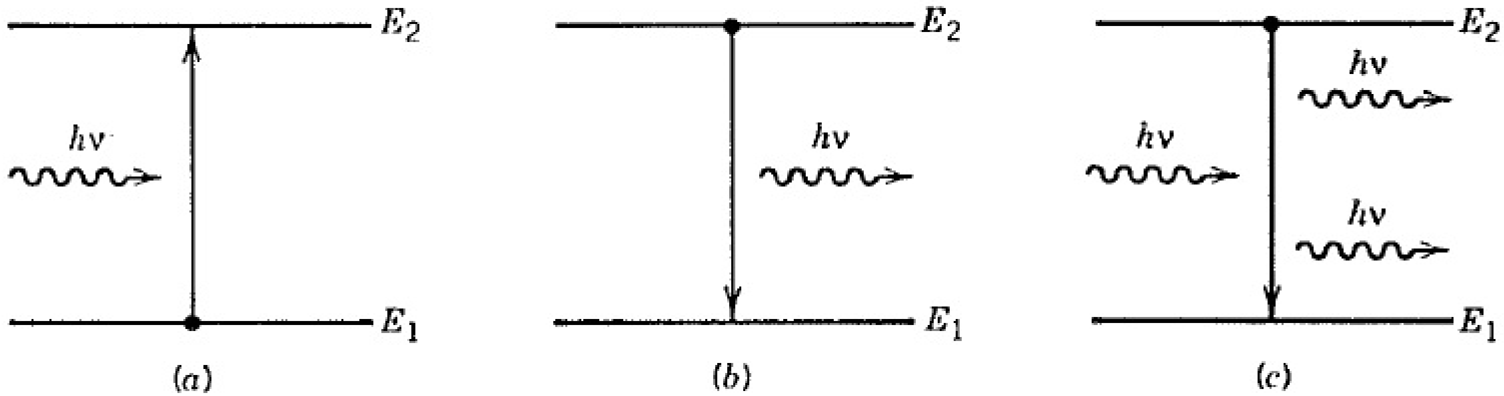
Legea lui Wien

$$h\nu \ll kT$$

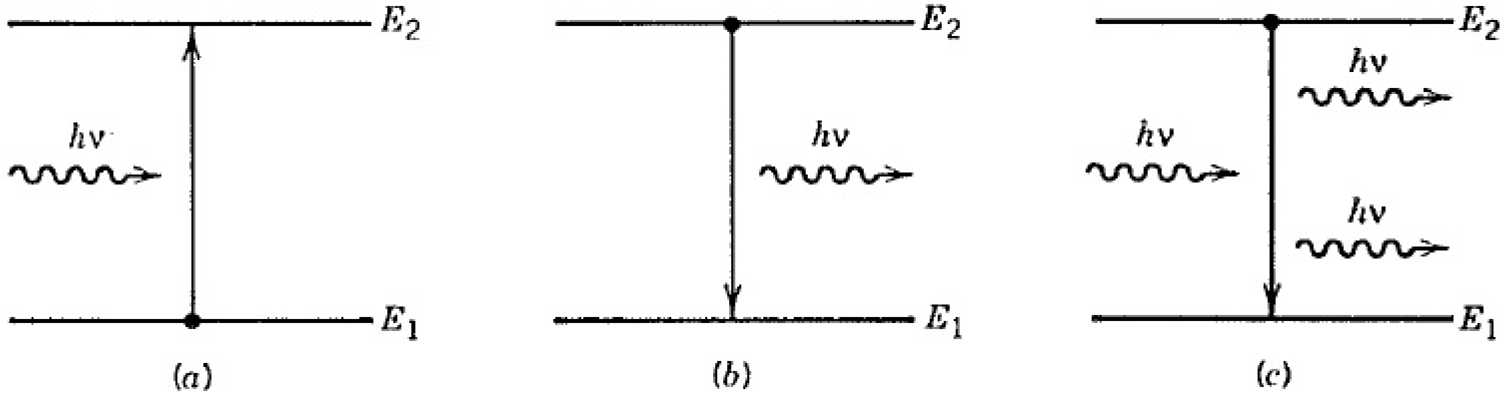
$$e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT}$$

Legea Rayleigh–Jeans

Absorbtia si emisia de lumina



Viteza de absorbtia si emisie



$$R_{spon} = AN_2 \left(36\right), \quad R_{stim} = BN_2\rho_{em} \left(37\right), \quad R_{abs} = B' N_1\rho_{em} \left(38\right)$$

$$N_2/N_1 = Exp\left(-E_g/kT\right) = Exp\left(-h\nu/kT\right) \quad \left(39\right)$$

$$AN_2 + BN_2\rho_{em} = B' N_1\rho_{em} \left(40\right) \Rightarrow \rho_{em} = \frac{A/B}{\left(B'/B\right)\exp\left(h\nu/kT\right)-1}$$

$$\rho_{em} = \frac{8\pi h\nu^3/c^3}{\exp\left(h\nu/kT\right)-1} \left(41\right) \Rightarrow \boxed{A = \left(8\pi h\nu^3/c^3\right)B \; si \; B' = B \quad \left(42\right)}$$

Concluzii

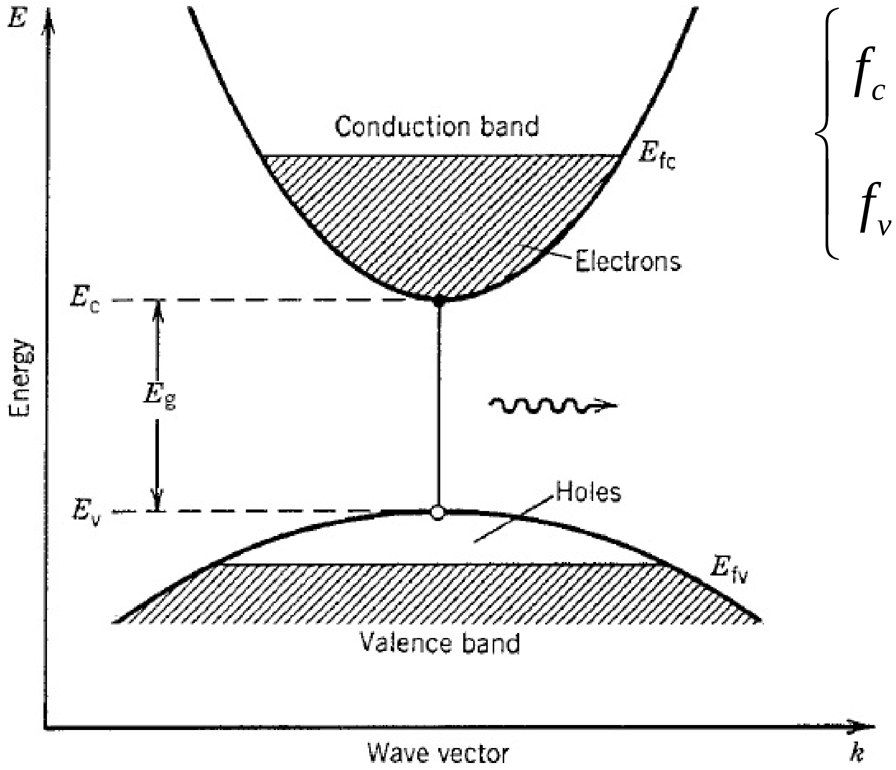
1. Sursele termice

2. Echilibru termic la temperature camerei

$$R_{stim}/R_{spon} = \left[\exp\left(h\nu/kT\right)-1\right]^{-1} \ll 1 \quad (43)$$

Laserele functioneaza prin inversiunea de populatie N2 > N1

Absorbtia si emisia in semiconductori

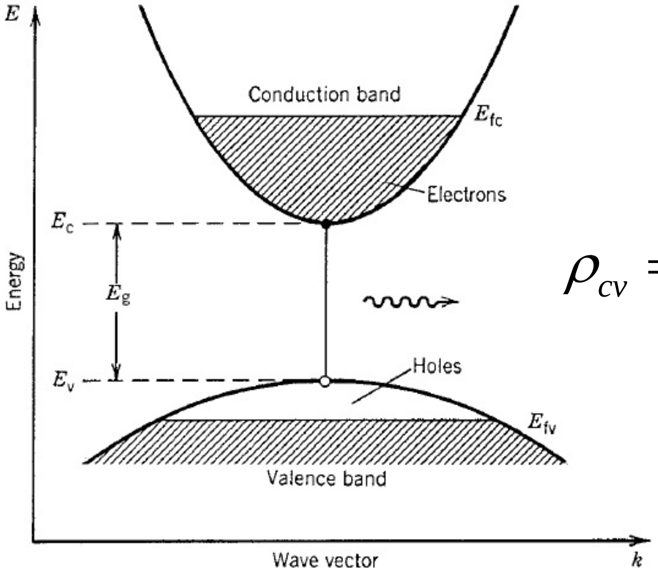


$$\begin{cases} f_c(E_2) = \left\{1 + \exp\left[\left(E_2 - E_{fc}\right)/kT\right]\right\}^{-1} \\ f_v(E_1) = \left\{1 + \exp\left[\left(E_1 - E_{fv}\right)/kT\right]\right\}^{-1} \end{cases} \quad (44)$$

$$E_2 - E_1 = E_{em} = \hbar\omega$$

$$\hbar = h/2\pi$$

Absorbtia si emisia in semiconductori - 2



$$\rho_{cv} = \frac{\left(2m_r\right)^{3/2}}{2\pi^2\hbar^3}\left(\hbar\omega - E_g\right)^{1/2},\ m_r = m_c m_v / \left(m_c + m_v\right) \quad (45)$$

$$R_{spon}\left(\omega\right) = \int_{E_c}^{\infty} A\left(E_1, E_2\right) f_c\left(E_2\right)\left[1 - f_v\left(E_1\right)\right]\rho_{cv} dE_2 \quad (46)$$

$$R_{stim}\left(\omega\right) = \int_{E_c}^{\infty} B\left(E_1, E_2\right) f_c\left(E_2\right)\left[1 - f_v\left(E_1\right)\right]\rho_{cv}\rho_{em} dE_2 \quad (47)$$

$$R_{abs}\left(\omega\right) = \int_{E_c}^{\infty} B\left(E_1, E_2\right) f_v\left(E_1\right)\left[1 - f_c\left(E_2\right)\right]\rho_{cv}\rho_{em} dE_2 \quad (48)$$

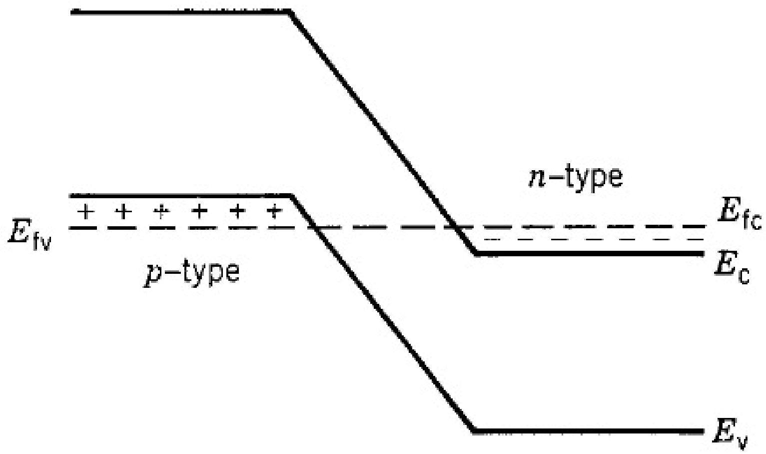
Inversiunea de populatie

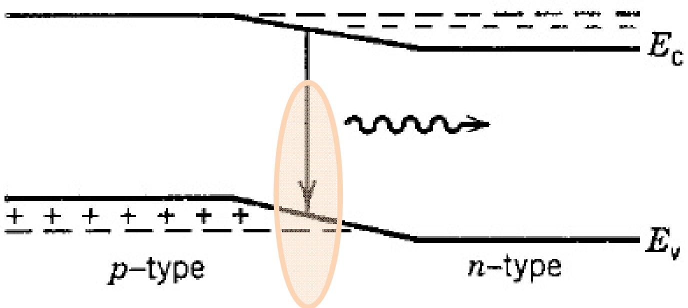
$$R_{stim} > R_{abs} \qquad (49)$$

$$f_c(E_2) > f_v(E_1)$$

$$E_{fc} - E_{fv} > E_2 - E_1 > E_g$$

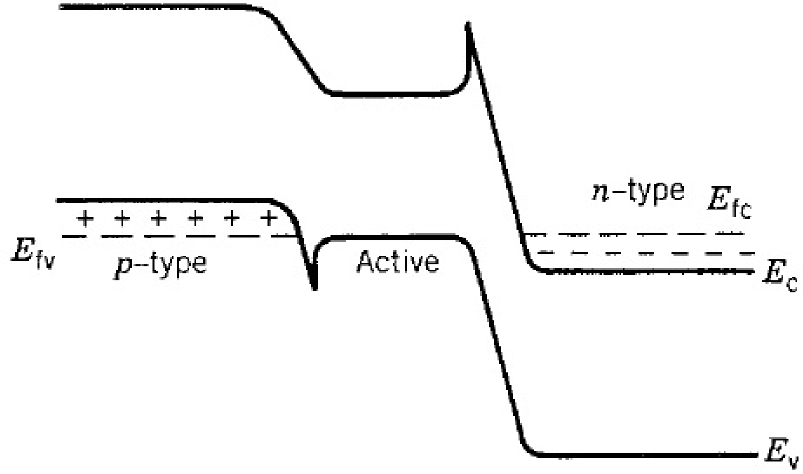
Homo-jonctiunea p-n



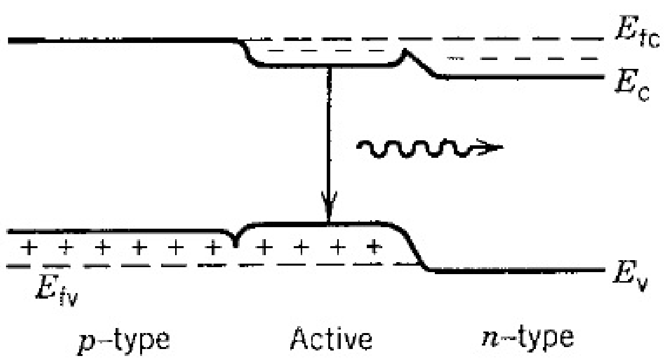


$$I = I_S\left[\exp\left(qV/kT\right)-1\right] \quad (50)$$

Hetero-jonctiunea p-n

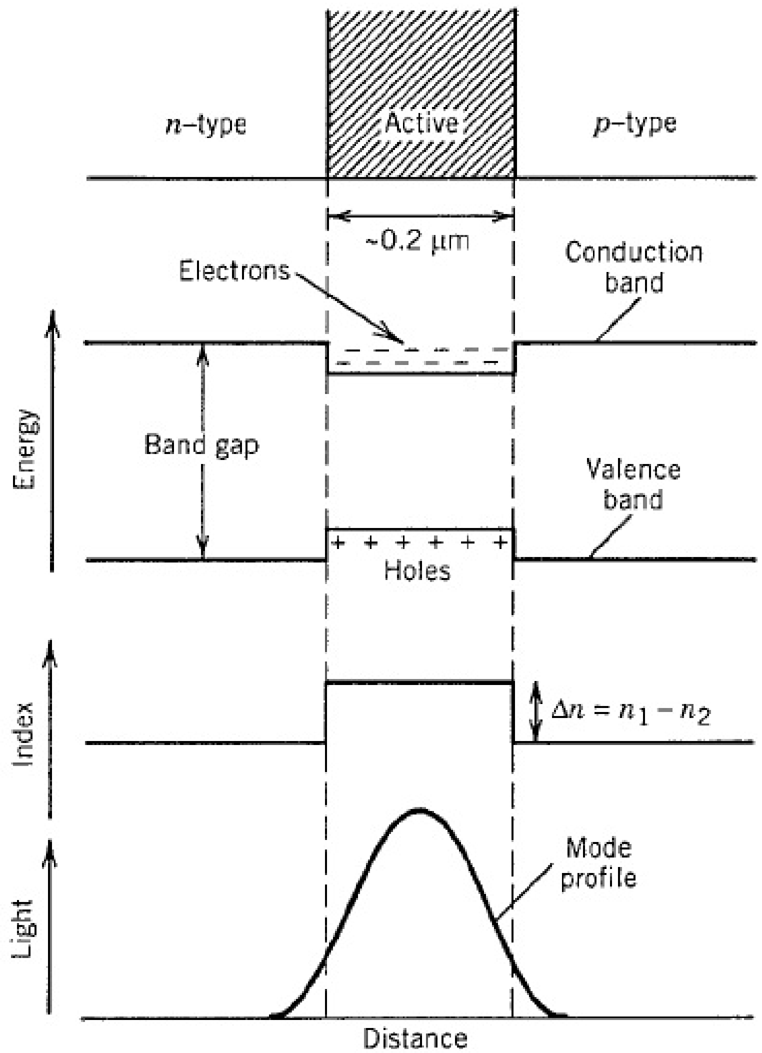


Nepolarizat

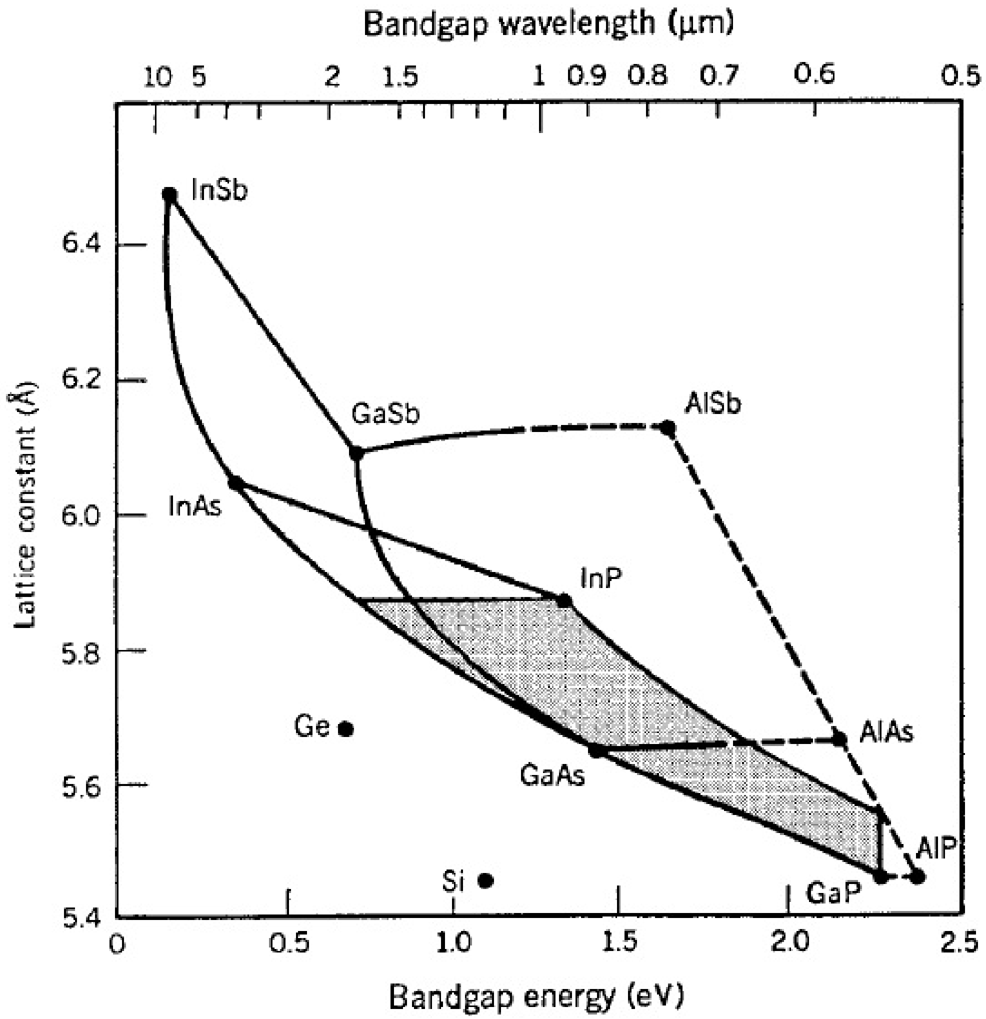


Polarizat direct

Confinarea simultana a purtatorilor si cimpului



Materiale semiconductoare



$$E_g\left(x\right)=1.424+1.247x,\left(0<x<0.45\right) \quad (51)$$

$$E_g\left(y\right)=1.35-0.72y+0.12y^2,\left(0<y<1\right) \quad (52)$$

Jonctiune p-n

$$\eta_{\text{int}} = \frac{R_{rr}}{R_{tot}} = \frac{R_{rr}}{R_{rr} + R_{nr}} \ (53)$$

$$R_{rr} = \frac{N}{\tau_{rr}}, R_{nr} = \frac{N}{\tau_{nr}} \ (54) \rightarrow \qquad \eta_{\text{int}} = \frac{\tau_{nr}}{\tau_{rr} + \tau_{nr}} \ (55)$$

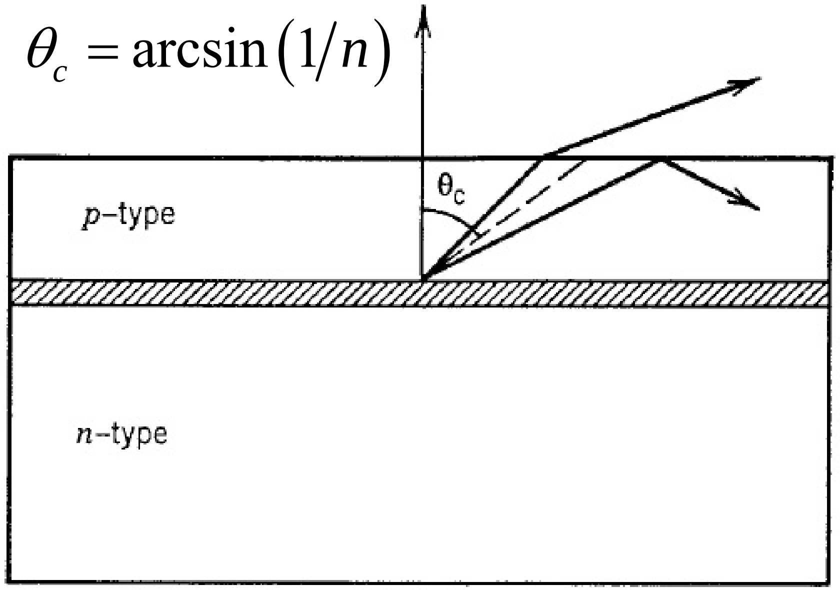
$$R_{rr} = R_{spon} + R_{stim} \ (56)$$

$$R_{spon} + R_{nr} = \frac{N}{\tau_c} \ (57)$$

$$\tau_c^{-1} = A_{nr} + BN + CN^2 \ (58)$$

LED

$$\theta_c = \arcsin(1/n)$$



$$P_{\text{int}} = \eta_{\text{int}}\left(\frac{\hbar\omega}{q}\right) I \ (59)$$

$$P_{ext} = \eta_{ext} P_{\text{int}} = \eta_{ext}\eta_{\text{int}}\left(\frac{\hbar\omega}{q}\right) I \ (60)$$

$$\eta_{ext} = \frac{1}{4\pi}\int_0^{\theta_c} T_f(\theta)(2\pi\sin\theta)\,d\theta \ (61)$$

$$S(\theta) = S_0\cos\theta \ (64)$$

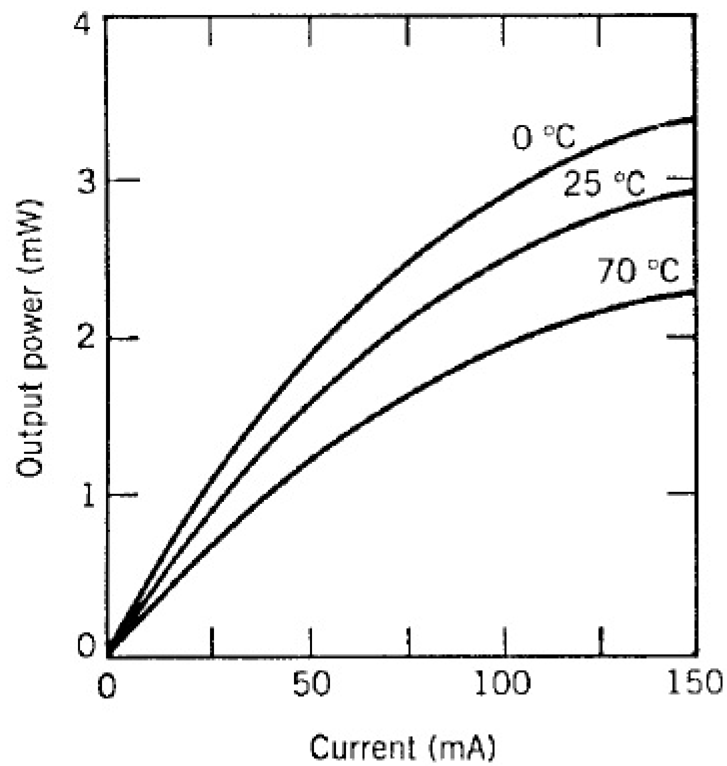
$$T_f(0) = 4n/(n+1)^2 \ (64$$

$$\eta_c = (NA)^2 \ (65)$$

$$\eta_{ext} = \frac{1}{n(n+1)^2} \ (63)$$

$$\eta_{tot} \stackrel{d}{=} \frac{P_e}{P_{elec}} = \frac{P_e}{V_0 I} = \eta_{ext}\eta_{\text{int}}\frac{\hbar\omega}{qV_0} \ (66) \xrightarrow{\hbar\omega\approx qV_0} \eta_{tot} \approx \eta_{ext}\eta_{\text{int}} \ (67)$$

Responzivitatea LED



$$R_{LED} = \frac{P_e}{I} = \eta_{ext}\eta_{\text{int}}\frac{\hbar\omega}{q} \quad (68)$$

Spectrul unui LED

$$R_{spon}(\omega) = A_0\sqrt{\hbar\omega - E_g}\,\exp\left[-\left(\hbar\omega - E_g\right)/kT\right] \quad (69)$$

$$\hbar\omega = E_g + \frac{kT}{2} \quad (70)$$

$$\Delta\nu = 1.8\frac{kT}{h} \quad (71)$$

$$\Delta\nu = \frac{1.8\cdot 1.38\cdot 10^{-23}\left(J/K\right)\cdot 300\left(K\right)}{6.626*10^{-34}\left(J\cdot s\right)} \approx 11.2THz$$

Raspunsul la modulatie al LED-ului

$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_c} \quad (72)$$

$$I(t) = I_b + I_m \exp(i\omega_m t) \quad (73)$$

$$N(t) = N_b + N_m \exp(i\omega_m t) \quad (74)$$

$$N_b = \tau_c I_b / qV \quad (75) \qquad N_m(\omega_m) = \frac{\tau_c I_m / qV}{1 + i\omega_m \tau_c} \quad (76)$$

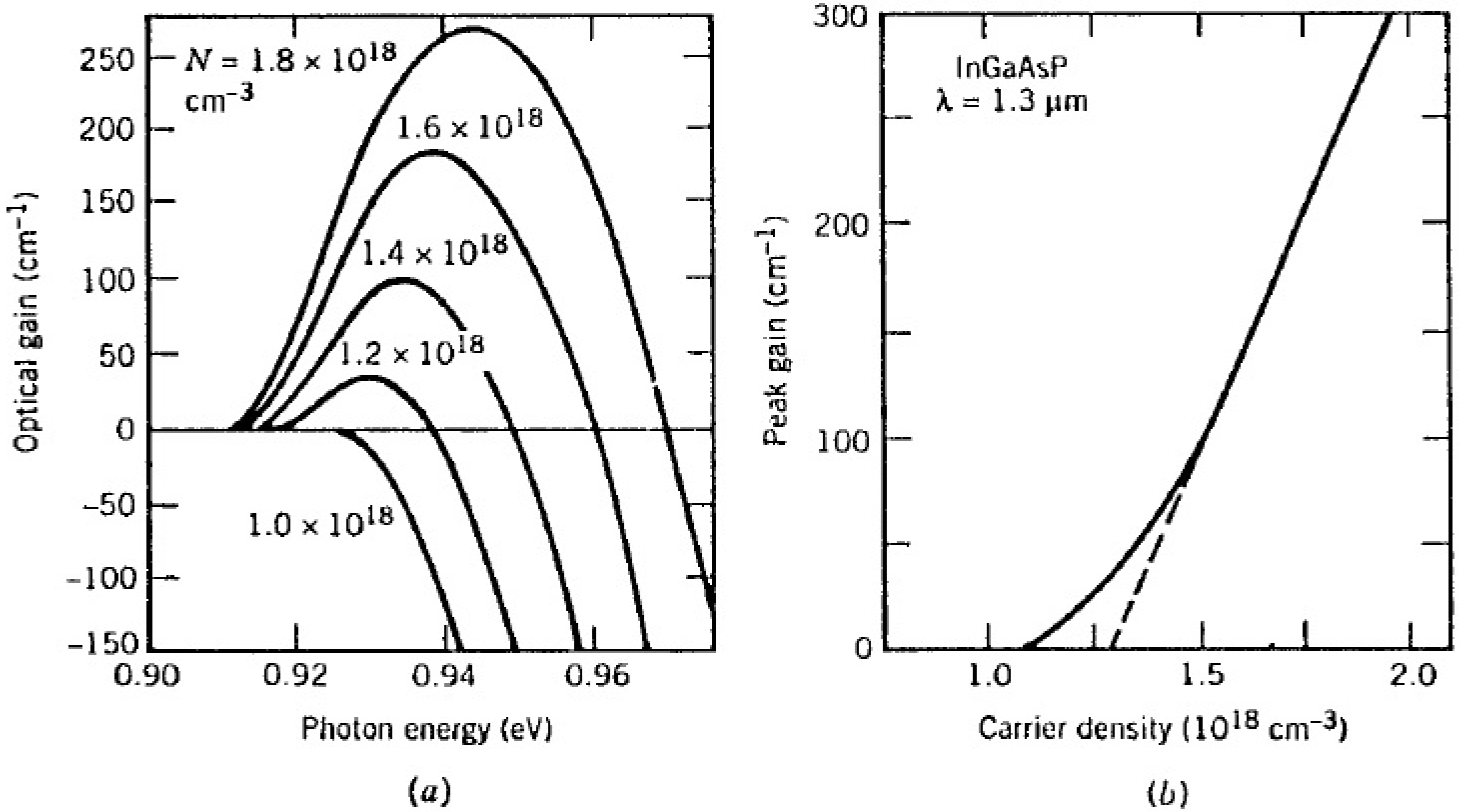
$$H(\omega_m) = \frac{N_m(\omega_m)}{N_m(0)} = \frac{1}{1 + i\omega_m \tau_c} \quad (77)$$

$$f_{3dB\_optic} = \frac{\sqrt{3}}{2\pi\tau_c} \quad (78) \quad f_{3dB\_electric} = \frac{1}{2\pi\tau_c} \quad (79)$$

Dioda LASER

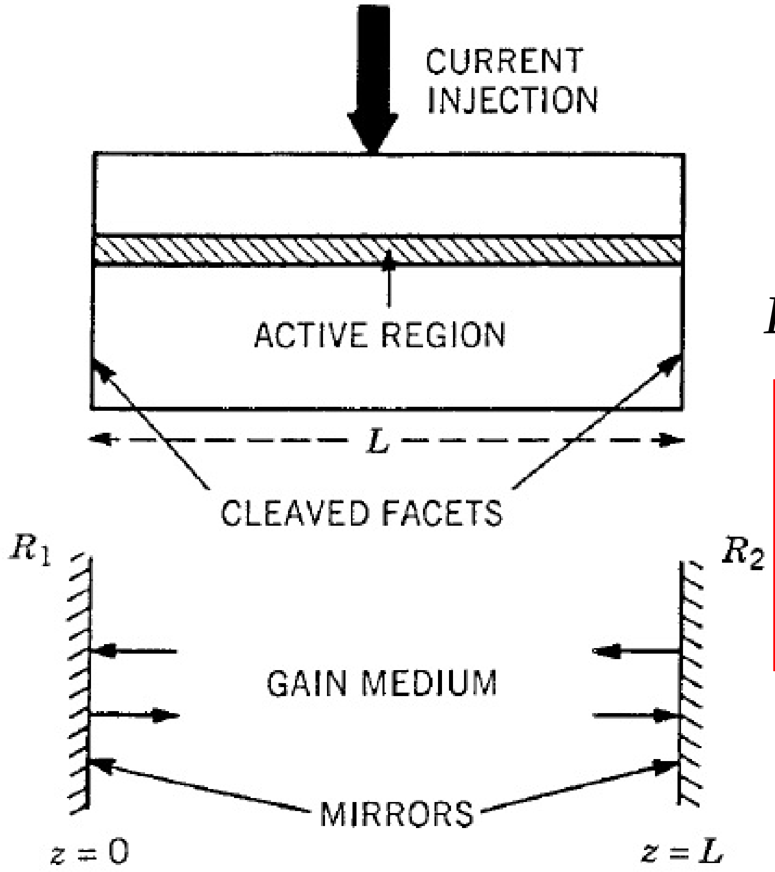
- Emit puteri mari (~100 mW)
- Emit lumina coerenta
- Fascicolul emis are o imprastiere unghiulara mica
- Latimea spectrului este mica
- Modulatia directa a diodei laser pina la frecvente mari (~ 25 GHz)

Cistigul optic



$$g_p(N) = \sigma_g (N - N_T) \quad (80)$$

Reactia si pragul LASER



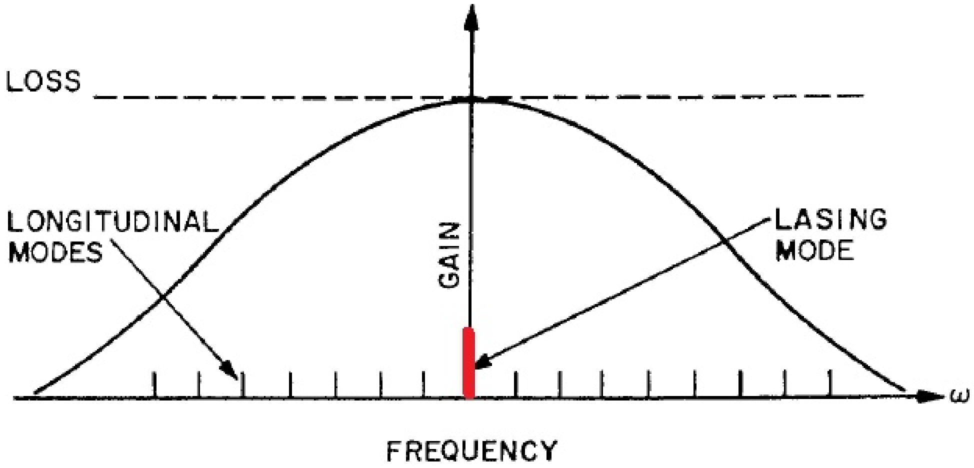
$$R_m = \left(\frac{n-1}{n+1}\right)^2 \quad (81)$$

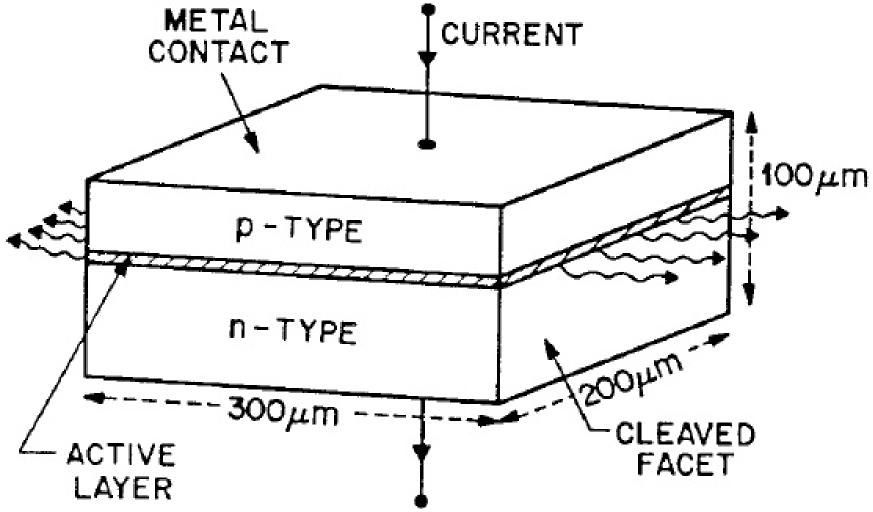
$$E_0 \exp(gL)\sqrt{R_1R_2}\exp(-\alpha_{\text{int}}L)\exp(2ikL) = E_0 \quad (82)$$

$$\begin{cases} g = \alpha_{\text{int}} + \dfrac{1}{2L}\ln\left(\dfrac{1}{R_1R_2}\right) = \alpha_{\text{int}} + \alpha_{mir} = E_0 \\ 2kL = 2m\pi \quad sau \quad \nu = \nu_m = mc/2nL \end{cases} \quad (83)$$

$$g = \Gamma g_m \quad (84)$$

Moduri longitudinale





$$\Delta\nu_L = c/2nL \quad (85)$$

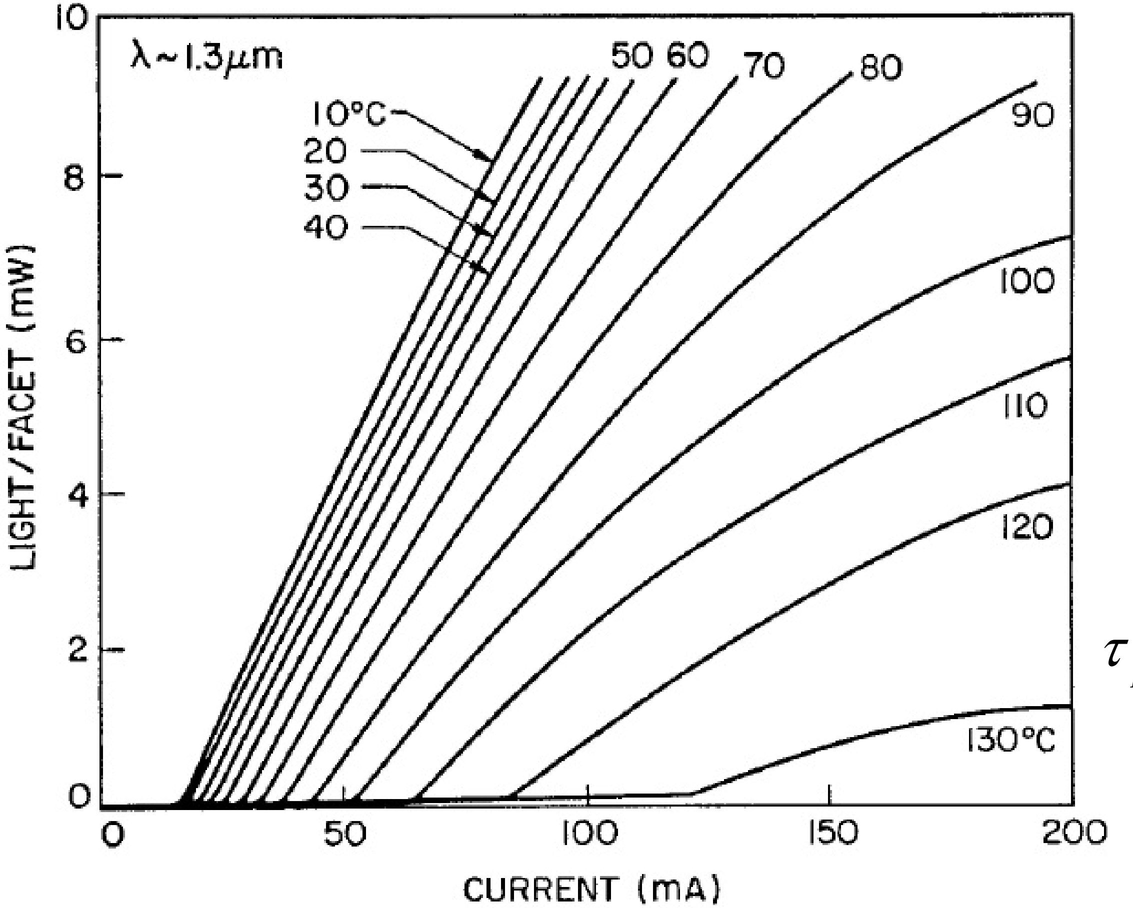
$$\Delta\nu_L = c/2n_g L \quad (86)$$

$$n_g = n + \omega\left(dn/d\omega\right) \quad (87)$$

Caracteristicile laser

- Caracteristici in unda continua
- Caracteristici de modulatie, la semnal mic
- Caracteristici de modulatie la semnal mare
- Zgomotul laserului

Caracteristicile laser in unde continua



$$\begin{cases} \frac{dP}{dt} = GP + R_{sp} - \frac{P}{\tau_p} & (88) \\ \frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - GP & (89) \end{cases}$$

$$G = \Gamma \mathrm{v}_g g_m = G_N \left(N - N_0\right) \quad (90)$$

$$G_N = \Gamma \mathrm{v}_g \, \sigma_g / V, \quad N_0 = N_T V$$

$$\tau_p^{-1} = \mathrm{v}_g \alpha_{cav} = \mathrm{v}_g \left(\alpha_{mir} + \alpha_{\mathrm{int}}\right) \quad (91)$$

$$I_{th}\left(T\right) = I_0 \exp\left(T/T_0\right) \quad (92)$$

Caracteristicile P-I a laserului

$$I_{th} = \frac{qN_{th}}{\tau_c} = \frac{q}{\tau_c}\left(N_0 + \frac{1}{G_N \tau_p}\right) \quad (93)$$

$$P = \left(\tau_p / q\right)\left(I - I_{th}\right), \quad I > I_{th} \quad (94)$$

$$P_e = \frac{1}{2}\left(\mathrm{v}_g \alpha_{mir}\right)\hbar\omega P \quad (95)$$

$$P_e = \frac{\hbar\omega}{2q}\frac{\eta_{\mathrm{int}}\alpha_{mir}}{\alpha_{mir} + \alpha_{\mathrm{int}}}\left(I - I_{th}\right) \quad (96)$$

$$\frac{dP_e}{dI} = \frac{\hbar\omega}{2q}\eta_d \quad cu \quad \eta_d = \frac{\eta_{\mathrm{int}}\alpha_{mir}}{\alpha_{mir} + \alpha_{\mathrm{int}}} \quad (97)$$

Eficienta cuantica externa a laserului

$$\eta_{ext} = \frac{rata\_emisie\_fotoni}{rata\_injectie\_electroni} = \frac{2P_e/\hbar\omega}{I/q} = \frac{2q}{\hbar\omega}\frac{P_e}{I} \quad (98)$$

$$\eta_{ext} = \eta_d\left(1 - \frac{I_{th}}{I}\right) \quad (99)$$

$$\eta_{tot} = \frac{2P_e}{V_0 I} \quad (100)$$

$$\eta_{tot} = \frac{\hbar\omega}{qV_0}\eta_{ext} \approx \frac{E_g}{qV_0}\eta_{ext} \quad (101)$$

Modulatia laserului

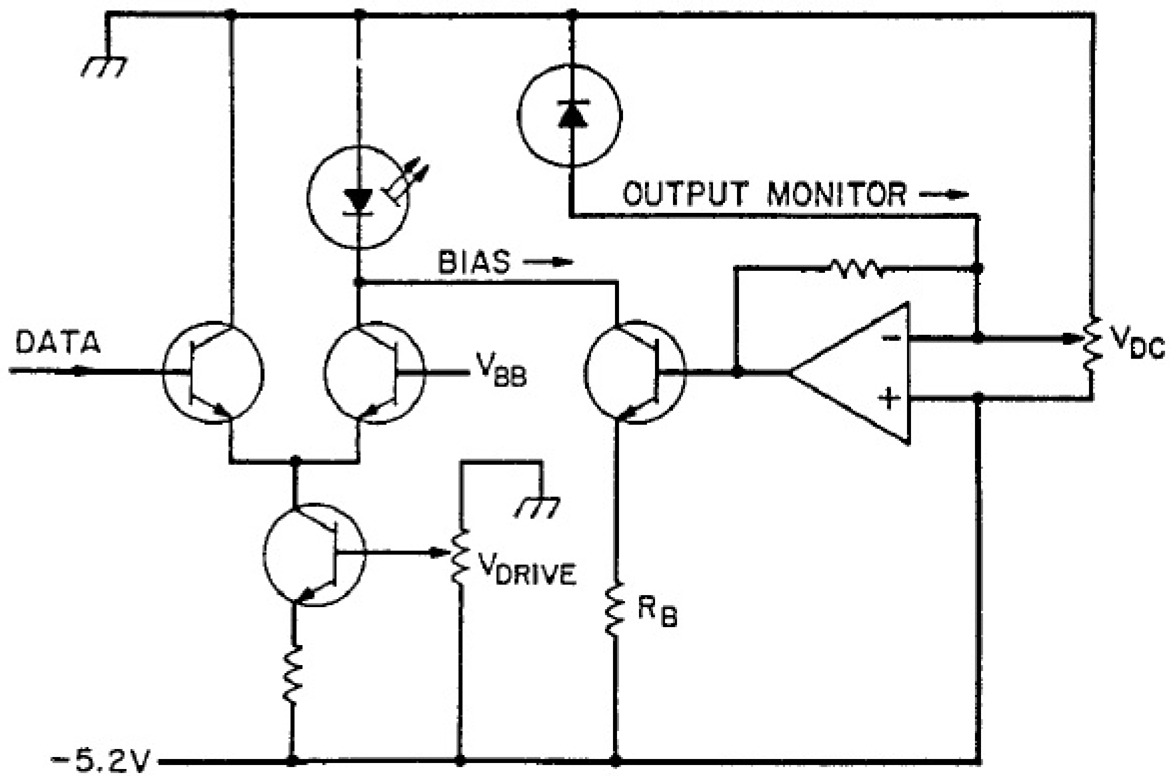
$$\begin{cases} \dfrac{dP}{dt} = GP + R_{sp} - \dfrac{P}{\tau_p} & (88) \\ \dfrac{dN}{dt} = \dfrac{I}{q} - \dfrac{N}{\tau_c} - GP & (89) \end{cases}$$

$$I(t) = I_b + I_m f_p(t) \quad (102)$$

$$G = G_N (N - N_0)(1 - \varepsilon_{NL} P) \quad (103)$$

$$\frac{d\phi}{dt} = \frac{1}{2}\beta_c \left[G_N (N - N_0) - \frac{1}{\tau_p} \right] \quad (104)$$

Modulatia laserului



Modulatia de semnal mic

$$I_b > I_{th}\ , I_m \ll I_b - I_{th}$$

$$f_p(t) = \sin(\omega_m t) \quad (105)$$

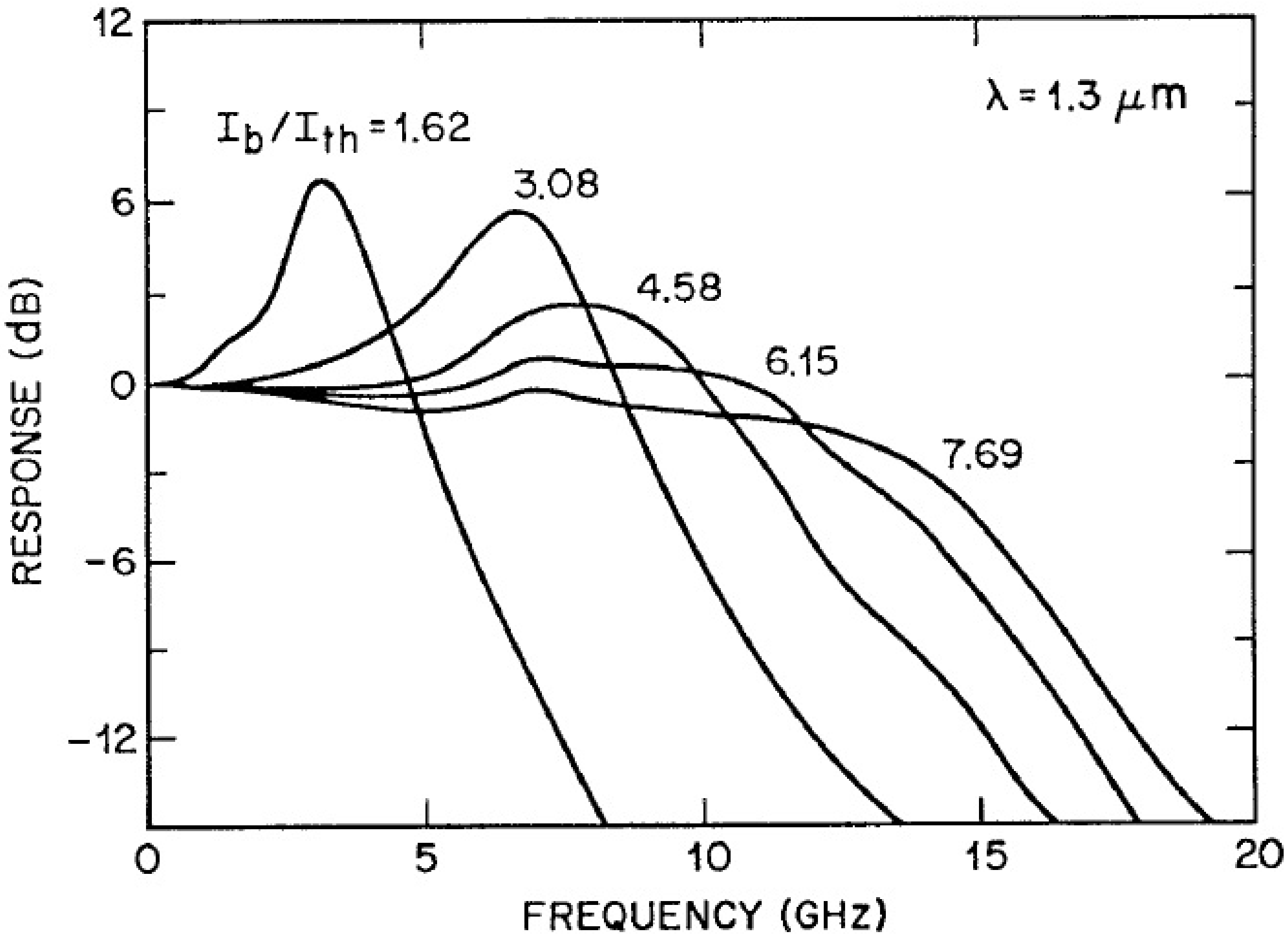
$$\begin{cases} P(t) = P_b + |p_m| \sin(\omega_m t + \theta_m) & (106) \\ N(t) = N_b + |n_m| \sin(\omega_m t + \psi_m) & (107) \end{cases}$$

$$p_m(\omega_m) = |p_m| \exp(i\theta_m) = \frac{P_b G_N\, I_m / q}{(\Omega_R + \omega_m - i\Gamma_R)(\Omega_R - \omega_m + i\Gamma_R)} \quad (108)$$

$$\Omega_R = \sqrt{G G_N P_b - (\Gamma_P - \Gamma_N)^2 / 4}\ , \Gamma_R = (\Gamma_P - \Gamma_N)/2 \quad (109)$$

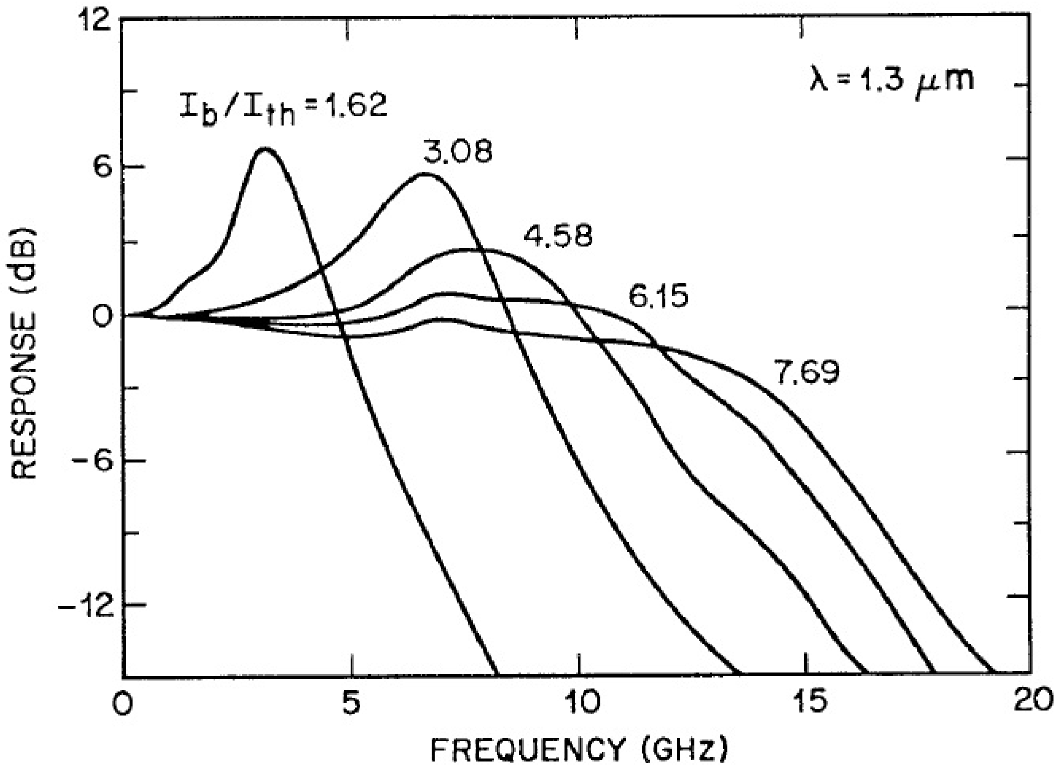
$$\Gamma_P = R_{sp} / P_b + \varepsilon_{NL} G P_b\ , \Gamma_N = \tau_c^{-1} + G_N P_b \quad (110)$$

Functia de transfer



$$H(\omega_m)=\frac{p_m(\omega_m)}{p_m(0)}=\frac{\Omega_R^2+\Gamma_R^2}{(\Omega_R+\omega_m-i\Gamma_R)(\Omega_R-\omega_m+i\Gamma_R)} \quad (111)$$

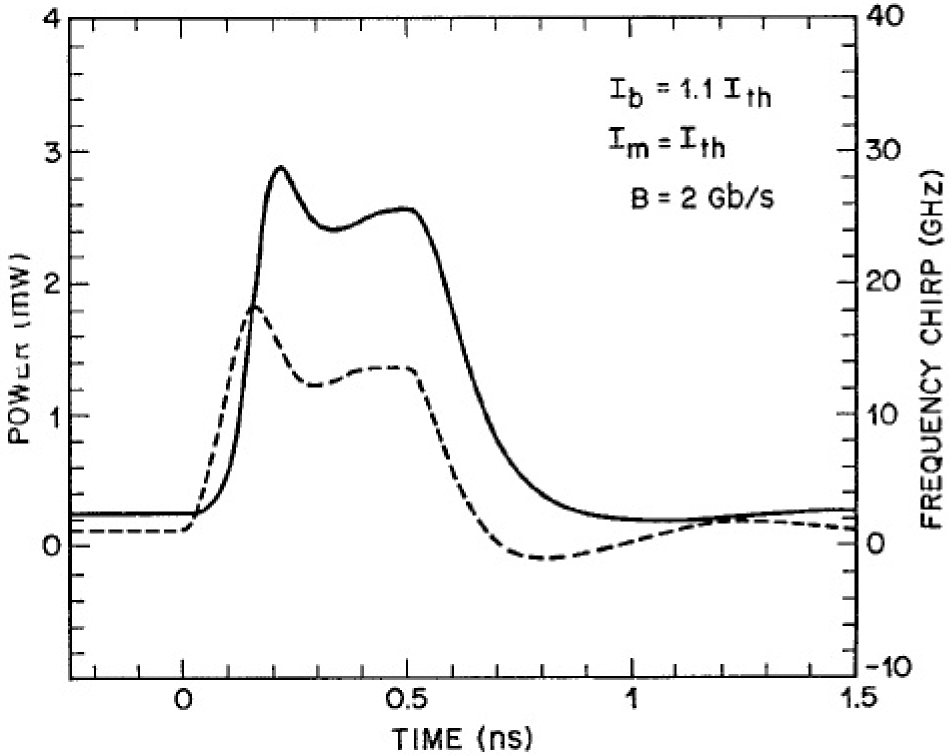
Banda de modulatie



$$f_{3dB} = \frac{1}{2\pi}\sqrt{\Omega_R^2 + \Gamma_R^2 + 2\left(\Omega_R^4 + \Omega_R^2\Gamma_R^2 + \Gamma_R^4\right)^{1/2}} \quad (112)$$

$$\Gamma_R \ll \Omega_R \Rightarrow f_{3dB} \approx \frac{\sqrt{3}\Omega_R}{2\pi} \approx \sqrt{\frac{3G_N P_b}{4\pi^2\tau_p}} = \sqrt{\frac{3G_N}{4\pi^2 q}\left(I_b - I_{th}\right)} \quad (113)$$

Modulatia de semnal mare



THz

$$\delta\nu(t) = \frac{1}{2\pi}\frac{d\phi}{dt} = \frac{\beta_c}{4\pi}\left[G_N\left(N - N_0\right) - \frac{1}{\tau_p}\right] \quad (114)$$

Zgomotul in dioda LASER

$$\begin{cases} \dfrac{dP}{dt} = GP + R_{sp} - \dfrac{P}{\tau_p} + F_P(t) \quad (115) \\ \dfrac{dN}{dt} = \dfrac{I}{q} - \dfrac{N}{\tau_c} - GP + F_N(t) \quad (116) \\ \dfrac{d\phi}{dt} = \dfrac{1}{2}\beta_c \left[G_N (N - N_0) - \dfrac{1}{\tau_p} \right] + F_\phi(t) \quad (117) \end{cases}$$

$$\left\langle F_i(t) F_j(t') \right\rangle = 2D_{ij}\delta(t - t') \quad (118)$$

$$D_{PP} = R_{sp}P \, , \, D_{\phi\phi} = R_{sp} / 4P \quad (119)$$

Zgomotul in dioda LASER - 2

$$C_{pp}(\tau) = \langle \delta P(t)\,\delta P(t+\tau) \rangle / \overline{P}^2 \quad (120)$$

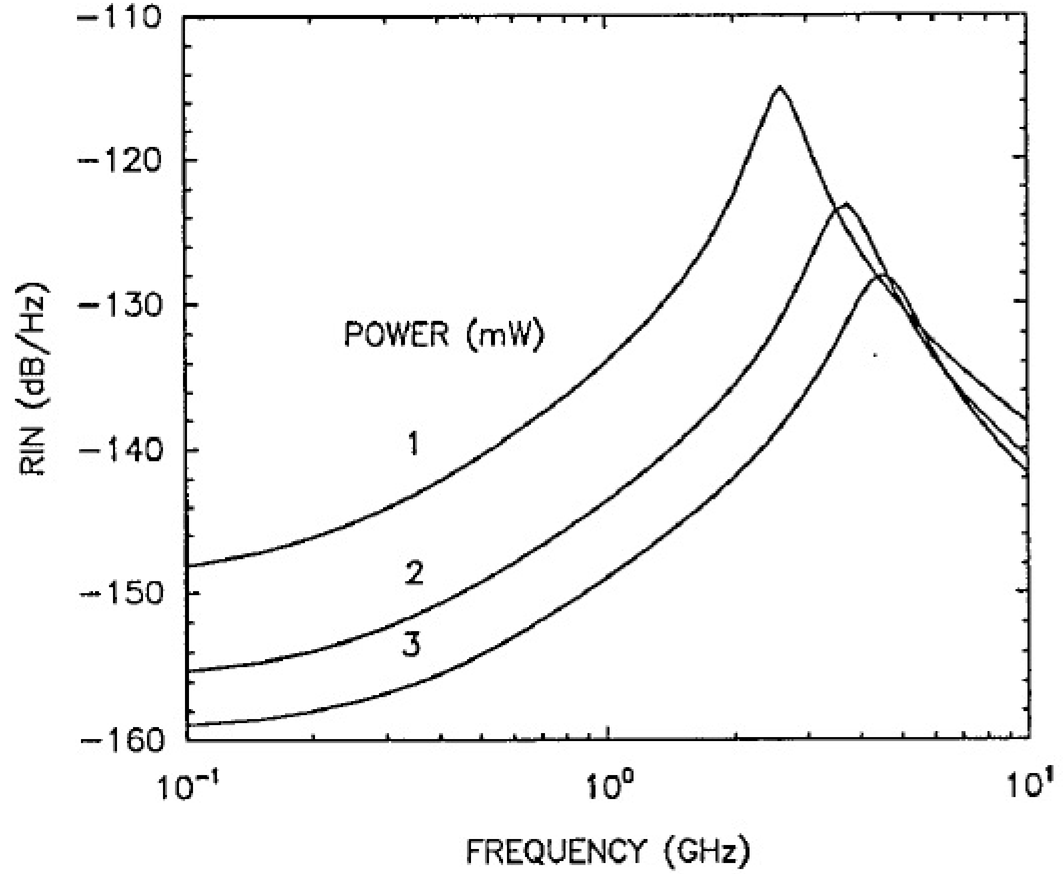
$$RIN(\omega) = \int_{-\infty}^{+\infty} C_{pp}(\tau) \exp(-i\omega t)\, dt \quad (121)$$

$$RIN(\omega) = \frac{2R_{sp}\left\{\left(\Gamma_N^2 + \omega^2\right) + G_N \overline{P}\left[G_N \overline{P}\left(1 + N/\tau_c R_{sp} \overline{P}\right) - 2\Gamma_N\right]\right\}}{\overline{P}\left[\left(\Omega_R - \omega\right)^2 + \Gamma_R^2\right]\left[\left(\Omega_R + \omega\right)^2 + \Gamma_R^2\right]} \quad (122)$$

$$\Omega_R = \sqrt{G G_N \overline{P} - \left(\Gamma_P - \Gamma_N\right)^2 / 4}\,,\ \Gamma_R = \left(\Gamma_P - \Gamma_N\right)/2 \quad (123)$$

$$\Gamma_R = R_{sp} / \overline{P} + \varepsilon_{NL} G \overline{P}\,,\ \Gamma_N = \tau_c^{-1} + G_N \overline{P} \quad (124)$$

Zgomotul in dioda LASER - 3



$$SNR \overset{d}{=} \frac{\overline{P}}{\sigma_p} = \frac{1}{\sqrt{C_{pp}(0)}} \quad (125)$$

$$SNR = \left(\frac{\varepsilon_{NL}}{R_{sp}\tau_p} \right)^{1/2} \overline{P} \quad (126)$$

Latimea liniei spectrale

$$S(\omega) = \int_{-\infty}^{\infty} \Gamma_{EE}(t) \exp\left[-i(\omega - \omega_0)\tau\right] d\tau \quad (127)$$

$$\Gamma_{EE}(t) = \left\langle E^*(t) E(t+\tau) \right\rangle, \; E(t) = \sqrt{P} \exp(i\phi) \quad (128)$$

$$\Gamma_{EE}(t) = \left\langle \exp\left[i\Delta\phi(t)\right] \right\rangle = \exp\left[-\left\langle \Delta\phi^2(\tau) \right\rangle \big/ 2\right] \quad (129)$$

$$\left\langle \Delta\phi^2(\tau) \right\rangle = \frac{R_{sp}}{2\overline{\overline{P}}} \left[\left(1 + \beta_c^2 b\right)\tau + \frac{\beta_c^2 b}{2\Gamma_R \cos\delta} \left[\cos(3\delta) - e^{-\Gamma_R \tau} \cos\left(\Omega_R \tau - 3\delta\right) \right] \right] \quad (130)$$

$$b = \Omega_R \Big/ \sqrt{\Omega_R^2 + \Gamma_R^2} \quad , \quad \delta = \arctan\left(\Gamma_R / \Omega_R\right) \quad (131)$$

$$\Delta\nu = R_{sp}\left(1 + \beta_c^2\right) \Big/ \left(4\pi \overline{P}\right) \quad (132)$$

Latimea liniei spectrale