Elements of Electromagnetics

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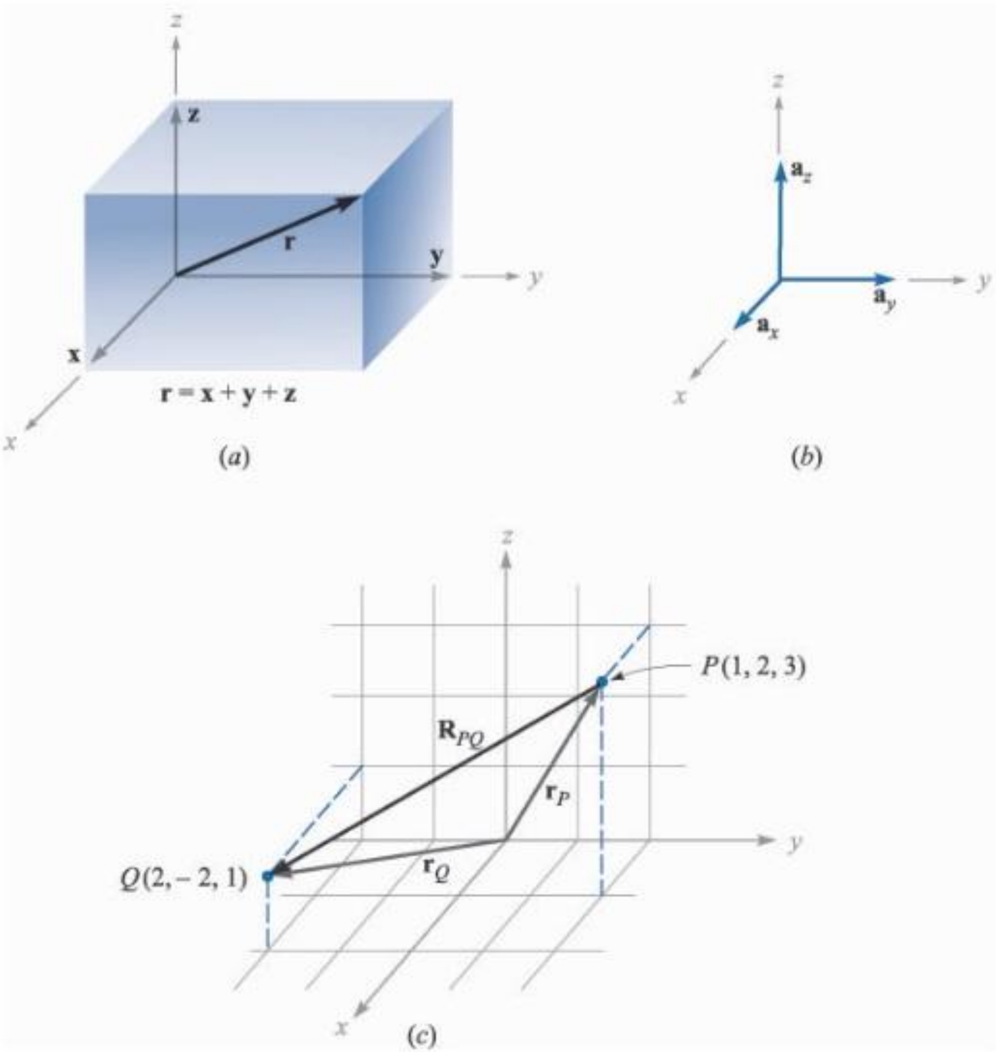

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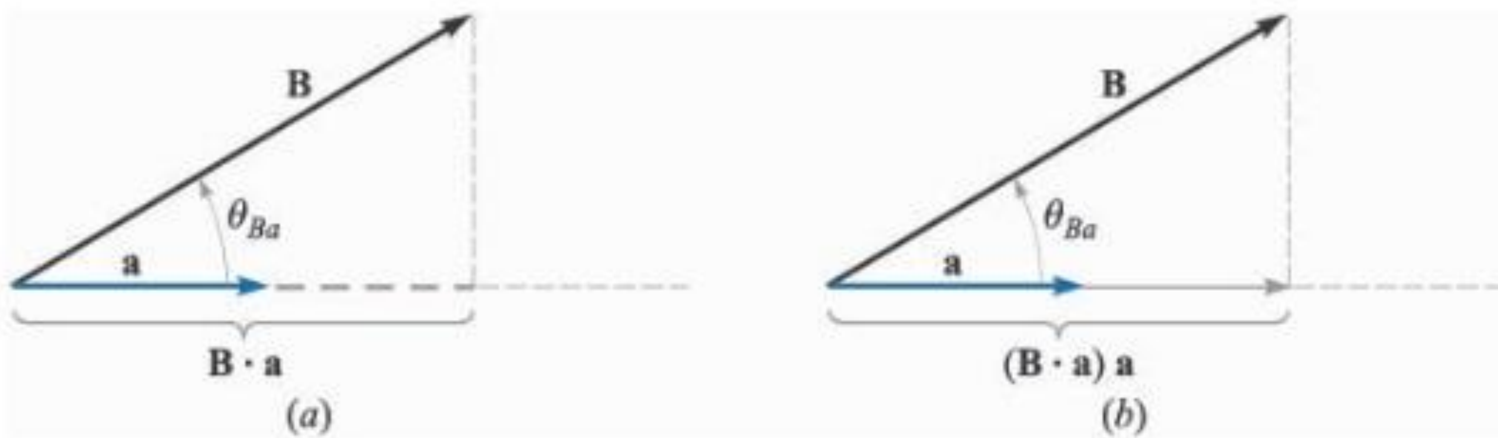

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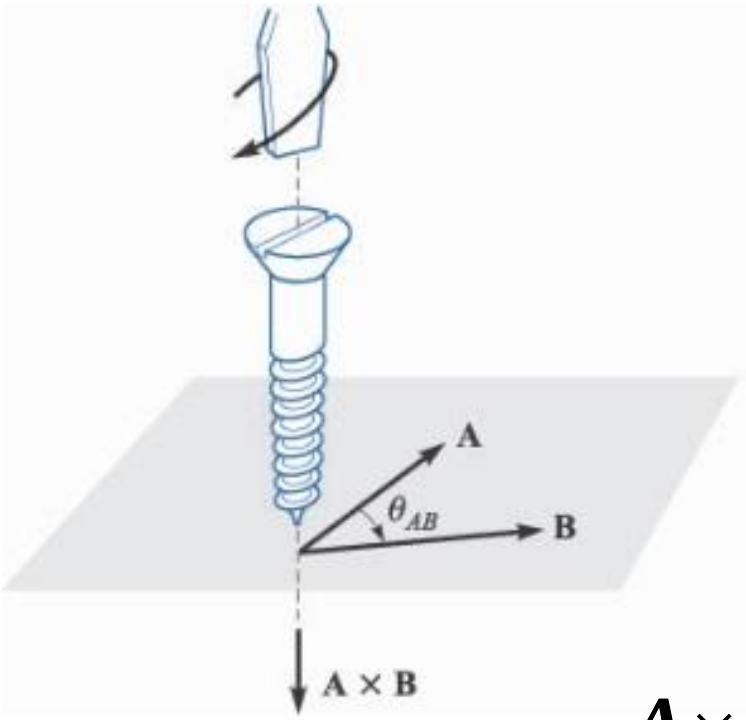

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

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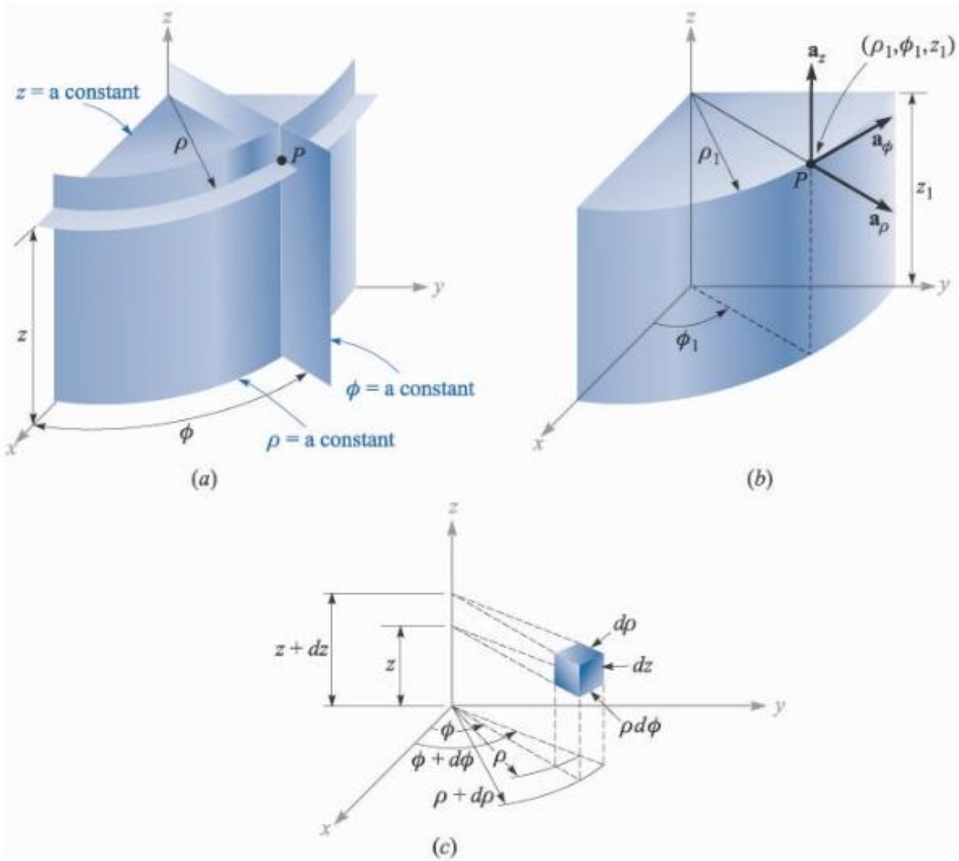
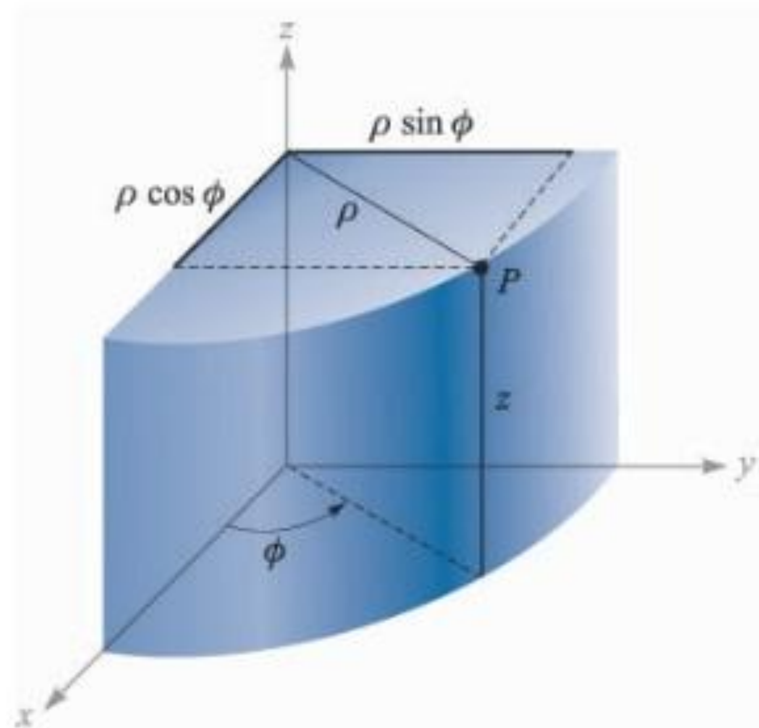
Sistemul de coordonate cilindrice circulare





$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = Arctg\left(\frac{y}{x}\right)$$
$$z = z$$

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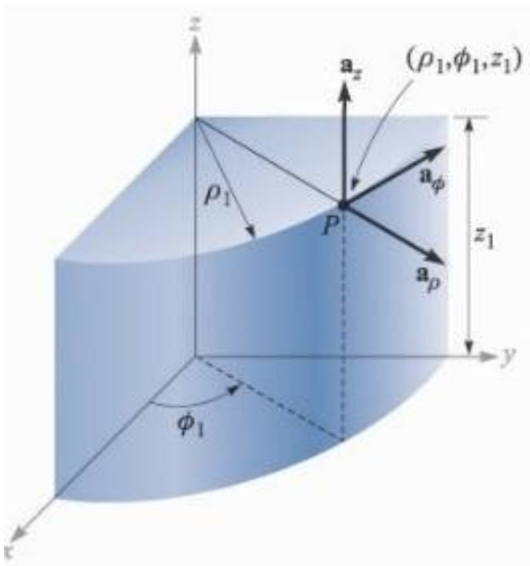
$$\boldsymbol{A} = A_\rho \boldsymbol{a}_\rho + A_\phi \boldsymbol{a}_\phi + A_z \boldsymbol{a}_z$$

$$A_\rho = \boldsymbol{A} \cdot \boldsymbol{a}_\rho = \left(A_x \boldsymbol{a}_x + A_y \boldsymbol{a}_y + A_z \boldsymbol{a}_z \right) \cdot \boldsymbol{a}_\rho = A_x \boldsymbol{a}_x \cdot \boldsymbol{a}_\rho + A_y \boldsymbol{a}_y \cdot \boldsymbol{a}_\rho + A_z \boldsymbol{a}_z \cdot \boldsymbol{a}_\rho$$

$$A_\phi = \boldsymbol{A} \cdot \boldsymbol{a}_\phi = \left(A_x \boldsymbol{a}_x + A_y \boldsymbol{a}_y + A_z \boldsymbol{a}_z \right) \cdot \boldsymbol{a}_\phi = A_x \boldsymbol{a}_x \cdot \boldsymbol{a}_\phi + A_y \boldsymbol{a}_y \cdot \boldsymbol{a}_\phi + A_z \boldsymbol{a}_z \cdot \boldsymbol{a}_\phi$$



(b)

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

Example 1.3

Transform the vector $\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$ into cylindrical coordinates.

Solution. The new components are

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$$
$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$
$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$
$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

Thus,

$$\mathbf{B} = -\rho\mathbf{a}_\phi + z\mathbf{a}_z$$

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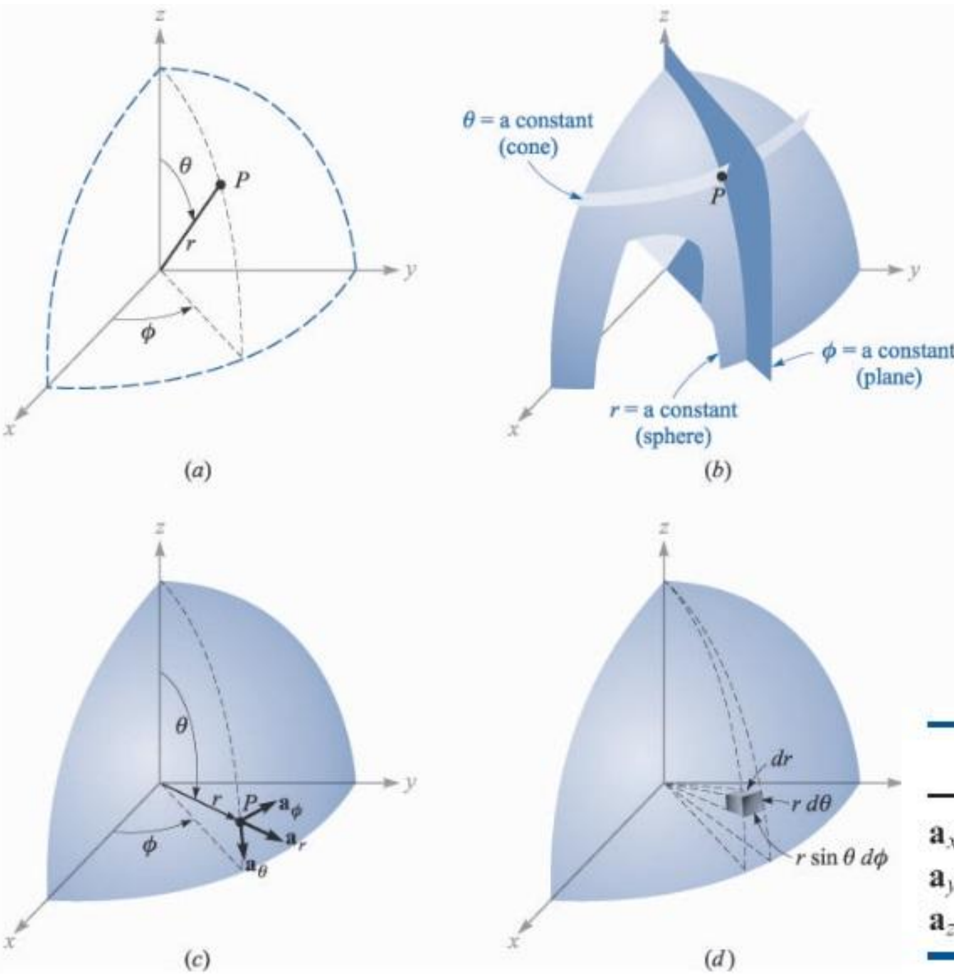
Sistemul de coordonate sferice



$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = Arc\cos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$
$$\phi = Arctg\left(\frac{y}{x}\right)$$

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

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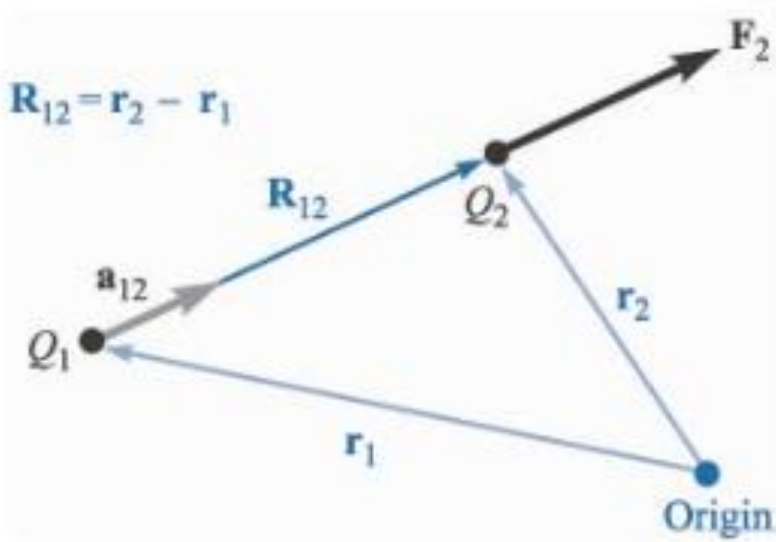
Forta Coulomb



$$\boldsymbol{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\boldsymbol{a}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\frac{\boldsymbol{R}_{12}}{R_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\frac{\boldsymbol{r}_2 - \boldsymbol{r}_1}{\left|\boldsymbol{r}_2 - \boldsymbol{r}_1\right|} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^3}\boldsymbol{R}_{12} \overset{n}{=} \boldsymbol{F}_{12}$$

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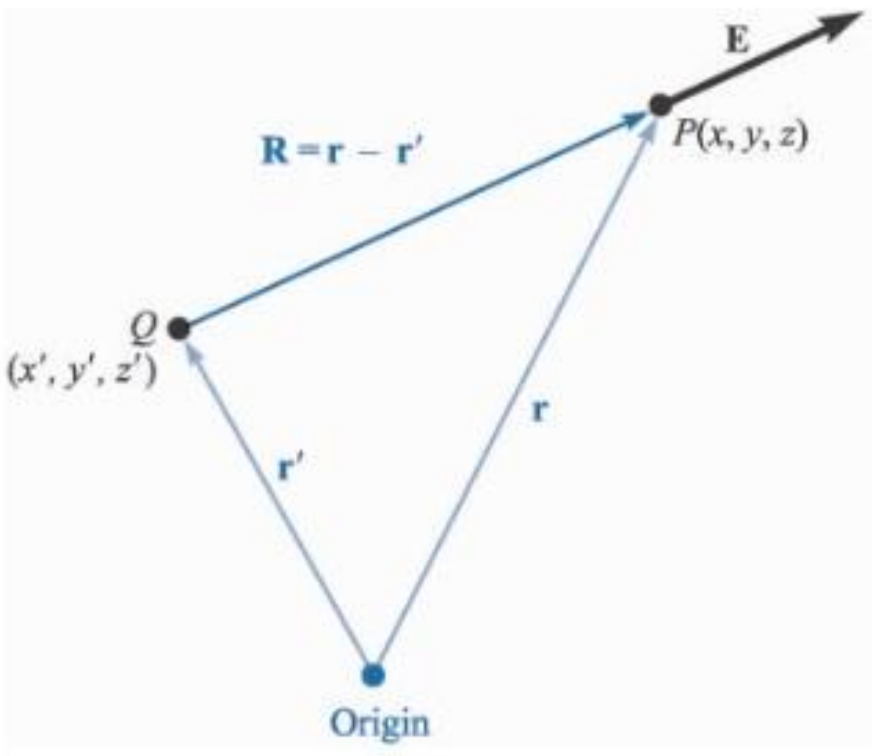
Cimpul Electric

$$\boldsymbol{E} = \frac{\boldsymbol{F}_{12}}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0 R_{12}^2}\boldsymbol{a}_{12} = \frac{Q_1}{4\pi\varepsilon_0 R_{12}^2}\frac{\boldsymbol{R}_{12}}{R_{12}} = \frac{Q_1}{4\pi\varepsilon_0 R_{12}^2}\frac{\boldsymbol{r}_2 - \boldsymbol{r}_1}{\left|\boldsymbol{r}_2 - \boldsymbol{r}_1\right|}$$



$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\boldsymbol{a}_R$$

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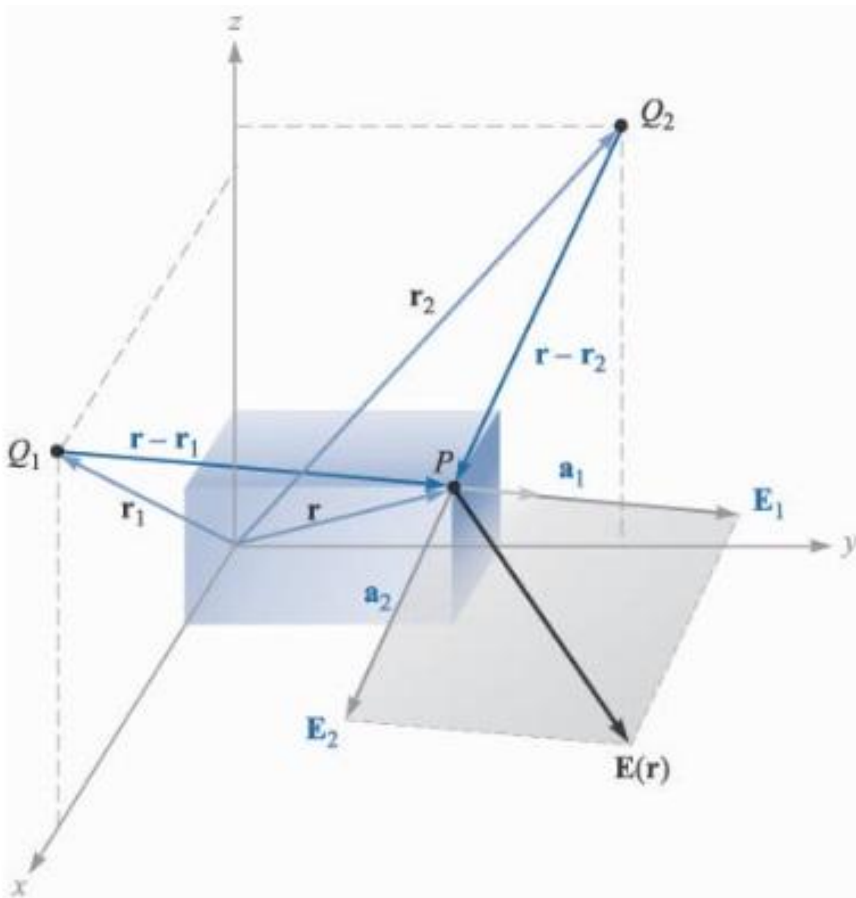
Sumarea cimpurilor electrice



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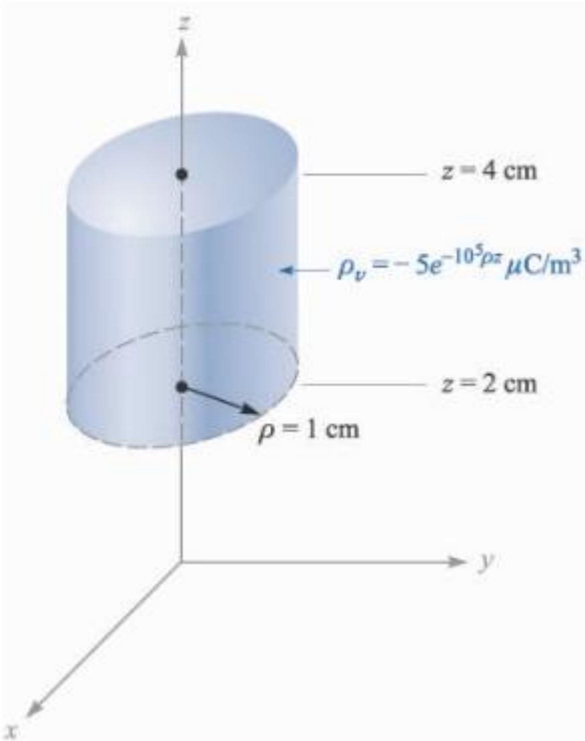
$$Q = \iiint_V \rho_v dv$$



$$\rho_v = -5\times10^{-6} e^{-10^5 \rho z}\ C/m^2$$

$$Q = \int_{0.02}^{0.04} \int_{0}^{2\pi} \int_{0}^{0.01} -5\times10^{-6} e^{-10^5 \rho z} \rho d\rho d\phi dz$$

$$Q = \int_{0.02}^{0.04} \int_{0}^{0.01} -10^{-5} \pi e^{-10^5 \rho z} \rho d\rho dz$$

$$Q = \int_{0}^{0.01} \left(\frac{-10^{-5}\pi}{-10^5 \rho} e^{-10^5 \rho z} \rho d\rho \right)_{z=0.02}^{z=0.04} = \int_{0}^{0.01} -10^{-10} \pi \left(e^{-2000\rho} - e^{-4000\rho} \right) d\rho$$

$$Q = -10^{-10} \pi \left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{-4000} \right)_{0}^{0.01} = -10^{-10} \pi \left(\frac{1}{2000} - \frac{1}{4000} \right) = \frac{-\pi}{40} = -0.0785 pC$$

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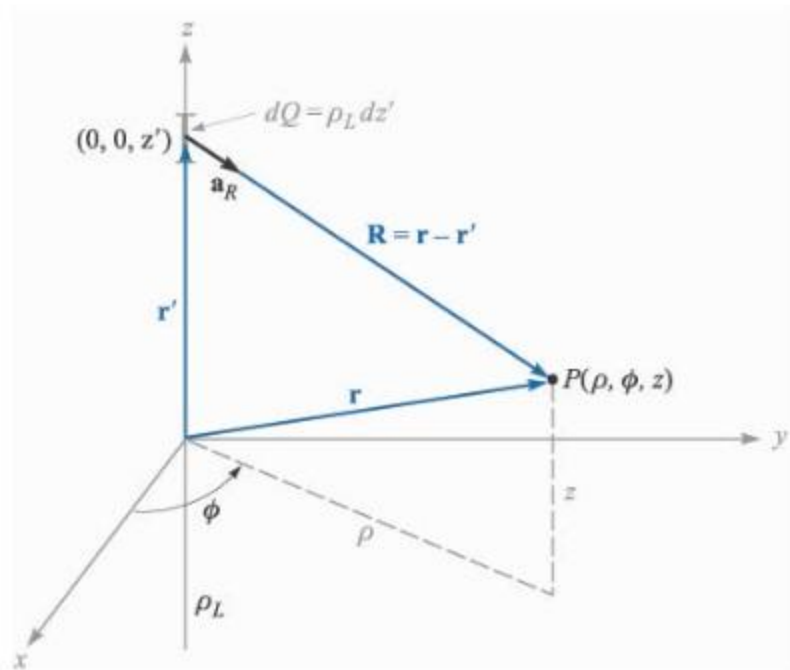
Cimpul unei sarcini liniare

$$\boldsymbol{E} = \iiint_V \frac{\rho_v dv'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^3} \rightarrow \int_{-\infty}^{\infty} \frac{\rho_L dz'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^3}$$



$$\boldsymbol{r} = \rho\boldsymbol{a}_\rho + z\boldsymbol{a}_z$$

$$\boldsymbol{r}' = z'\boldsymbol{a}_z$$

$$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}' = \rho\boldsymbol{a}_\rho + (z-z')\boldsymbol{a}_z$$

$$R = \sqrt{\rho^2 + (z-z')^2}$$

$$\boldsymbol{a}_R = \frac{\rho\boldsymbol{a}_\rho + (z-z')\boldsymbol{a}_z}{\sqrt{\rho^2+(z-z')^2}}$$

$$\boldsymbol{E} = \int_{-\infty}^{\infty} \frac{\rho_L dz'(\rho\boldsymbol{a}_\rho + (z-z')\boldsymbol{a}_z)}{4\pi\varepsilon_0 \left(\rho^2+(z-z')^2\right)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left\{\boldsymbol{a}_\rho \int_{-\infty}^{\infty} \frac{\rho dz'}{\left(\rho^2+(z-z')^2\right)^{3/2}} + \boldsymbol{a}_z \int_{-\infty}^{\infty} \frac{(z-z')dz'}{\left(\rho^2+(z-z')^2\right)^{3/2}}\right\}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left\{\left[\boldsymbol{a}_\rho \rho \frac{1}{\rho^2}\frac{-(z-z')}{\sqrt{\rho^2+(z-z')^2}}\right]_{-\infty}^{\infty} + \left[\boldsymbol{a}_z \frac{1}{\sqrt{\rho^2+(z-z')^2}}\right]_{-\infty}^{\infty}\right\}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left[\boldsymbol{a}_\rho \frac{2}{\rho} + \boldsymbol{a}_z 0\right] = \frac{\rho_L}{2\pi\varepsilon_0\rho}\boldsymbol{a}_\rho$$

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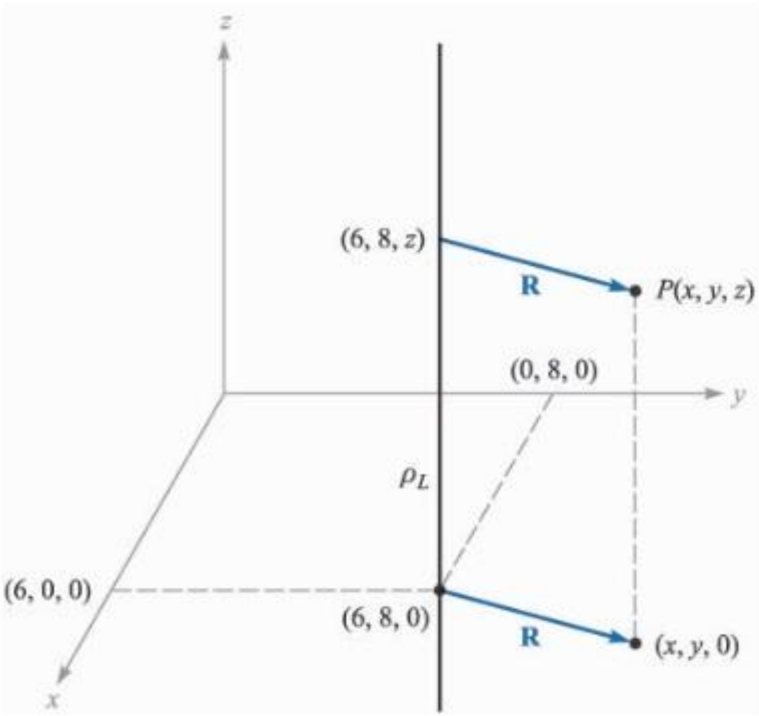
Exemplu



$$R = \sqrt{(x-6)^2 + (y-8)^2}$$

$$\boldsymbol{a}_\rho = \boldsymbol{a}_R = \frac{\boldsymbol{R}}{|\boldsymbol{R}|} = \frac{(x-6)\boldsymbol{a}_x + (y-8)\boldsymbol{a}_y}{\sqrt{(x-6)^2 + (y-8)^2}}$$

$$\boldsymbol{E} = \frac{\rho_L}{2\pi\varepsilon_0} \frac{(x-6)\boldsymbol{a}_x + (y-8)\boldsymbol{a}_y}{\sqrt{(x-6)^2 + (y-8)^2}}$$

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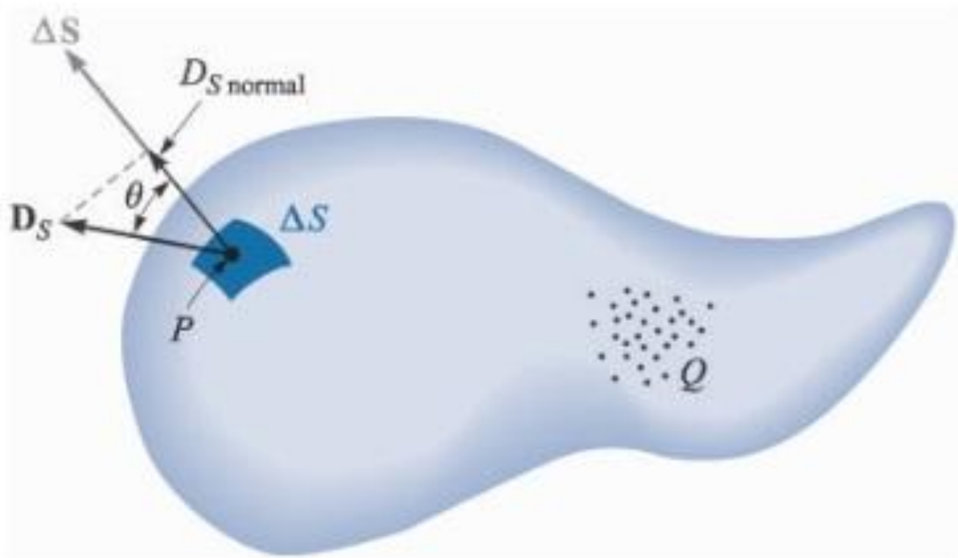
Flux electric



$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon_r \varepsilon_0 \boldsymbol{E}$$

$$\Psi = \oiint_S d\Psi = \oiint_S \boldsymbol{D}_S \cdot d\boldsymbol{S}$$

Legea lui Gauss

$$Q = \Psi$$

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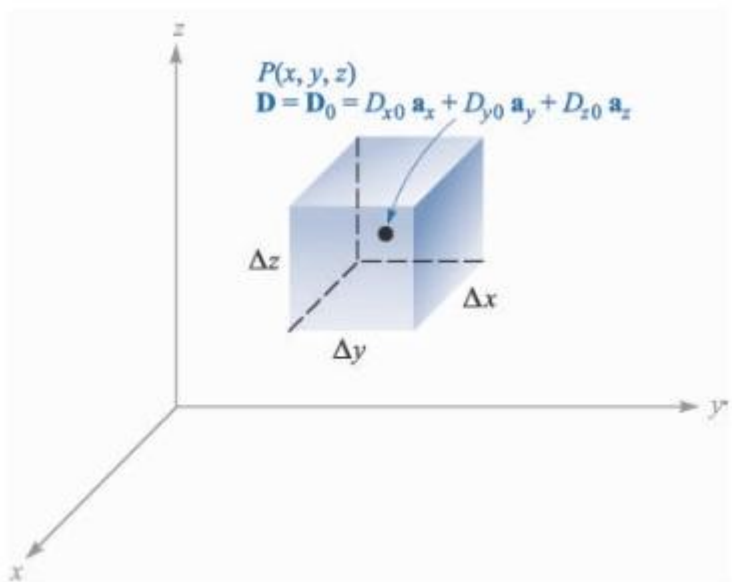
Divergenta



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