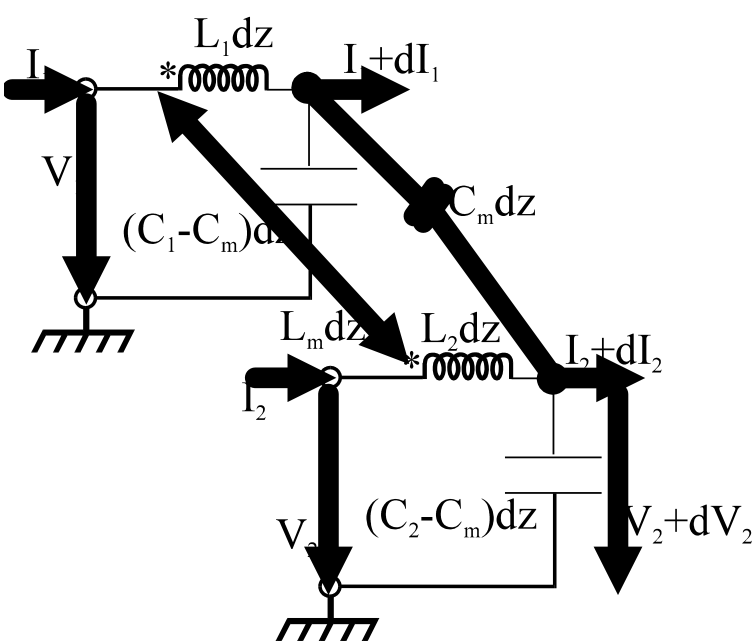
LINII CUPLATE

Ecuaţiile diferenţiale ale liniilor cuplate



$$
\begin{cases}
-\dfrac{dV_1}{dz} = j\omega L_1 I_1 + j\omega L_m I_2 \\
-\dfrac{dV_2}{dz} = j\omega L_m I_1 + j\omega L_2 I_2 \\
-\dfrac{dI_1}{dz} = j\omega C_1 V_1 - j\omega C_m V_2 \\
-\dfrac{dI_2}{dz} = -j\omega C_m V_1 + j\omega C_2 V_2
\end{cases}
$$

Solutia ecuatiilor diferentiale

$$\left\{\frac{d^2}{dz^2} - (j\omega)^2[L][C]\right\}[V] = 0 \qquad [C][L] = ([L][C])^T = \begin{bmatrix} L_1C_1 - L_mC_m & C_1L_m - C_mL_2 \\ -C_mL_1 + C_2L_m & -L_mC_m + L_2C_2 \end{bmatrix}$$

$$V = V_{0\cdot} \cdot e^{-\gamma z}$$

$$\begin{cases} a_1 = (L_mC_m - L_1C_1)\omega^2 \\ b_1 = (L_1C_m - L_mC_2)\omega^2 \\ a_2 = (L_mC_m - L_2C_2)\omega^2 \\ b_2 = -(L_mC_1 - L_2C_m)\omega^2 \end{cases}$$

$$\boxed{\gamma^4 - (a_1 + a_2)\gamma^2 + (a_1a_2 - b_1b_2) = 0}$$

Solutia ecuatiilor diferentiale-2

$$\gamma^4 - (a_1 + a_2)\gamma^2 + (a_1 a_2 - b_1 b_2) = 0$$

$$\gamma_\pm^2 = \frac{a_1 + a_2}{2} \pm \frac{1}{2}\sqrt{\left[(a_1 - a_2)^2 + 4b_1 b_2\right]}$$

$$\begin{cases} \gamma_+^2 \Rightarrow \left(\dfrac{V_2}{V_1}\right)_+ = \dfrac{1}{2b_1}\left[(a_2 - a_1) + \sqrt{(a_2 - a_1)^2 + 4b_1 b_2}\right] = R_+ \\ \gamma_-^2 \Rightarrow \left(\dfrac{V_2}{V_1}\right)_- = \dfrac{1}{2b_1}\left[(a_2 - a_1) - \sqrt{(a_2 - a_1)^2 + 4b_1 b_2}\right] = R_- \end{cases}$$

$$V_1 = A_1 \cdot e^{-\gamma_+ z} + A_2 \cdot e^{\gamma_+ z} + A_3 \cdot e^{-\gamma_- z} + A_4 \cdot e^{\gamma_- z}$$
$$V_2 = R_+\left(A_1 \cdot e^{-\gamma_+ z} + A_2 \cdot e^{\gamma_+ z}\right) + R_-\left(A_3 \cdot e^{-\gamma_- z} + A_4 \cdot e^{\gamma_- z}\right)$$

Linii simetrice

$$L_1 = L_2 = L_0 \qquad C_1 = C_2 = C_0$$

$$a_1 = a_2 = \left(L_m C_m - L_0 C_0\right)\omega^2 \qquad \gamma_+^2 = a_1 + \sqrt{b_1 b_2} \overset{n}{=} \gamma_e^2$$

$$b_1 = b_2 = \left(L_0 C_m - L_m C_0\right)\omega^2 \qquad \gamma_-^2 = a_1 - \sqrt{b_1 b_2} \overset{n}{=} \gamma_o^2$$

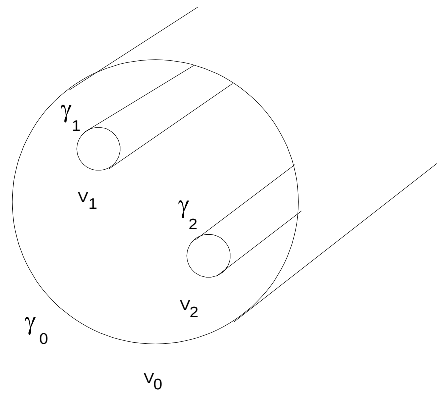
$$R_+ = \sqrt{\frac{b_2}{b_1}} = 1,\ R_- = -\sqrt{\frac{b_2}{b_1}} = -1$$

$$\begin{cases} V_1 = \left(A_1 \cdot e^{-\gamma_e z} + A_2 \cdot e^{\gamma_e z}\right) + \left(A_3 \cdot e^{-\gamma_o z} + A_4 \cdot e^{\gamma_o z}\right) \\ V_2 = \left(A_1 \cdot e^{-\gamma_e z} + A_2 \cdot e^{\gamma_e z}\right) - \left(A_3 \cdot e^{-\gamma_o z} + A_4 \cdot e^{\gamma_o z}\right) \\ I_1 = Y_{ce}\left(A_1 \cdot e^{-\gamma_e z} - A_2 \cdot e^{\gamma_e z}\right) + Y_{co}\left(A_3 \cdot e^{-\gamma_o z} - A_4 \cdot e^{\gamma_o z}\right) \\ I_2 = Y_{ce}\left(A_1 \cdot e^{-\gamma_e z} - A_2 \cdot e^{\gamma_e z}\right) - Y_{co}\left(A_3 \cdot e^{-\gamma_o z} - A_4 \cdot e^{\gamma_o z}\right) \end{cases}$$

$$\frac{j\omega\gamma_e (L_0 - L_m)}{D^2} = \frac{j\omega\gamma_e (L_0 - L_m)}{(j\omega)^2 \left(L_0^2 - L_m^2\right)} = \frac{-j\gamma_e}{(L_0 + L_m)\omega} \overset{n}{=} Y_{ce} =$$

$$\frac{j\omega\gamma_o (L_0 + L_m)}{D^2} = \frac{j\omega\gamma_o (L_0 + L_m)}{(j\omega)^2 (L_0^2 - L_m^2)} = \frac{-j\gamma_o}{(L_0 - L_m)\omega} \overset{n}{=} Y_{co} =$$

Consideratii electromagnetice asupra liniilor cuplate



$$V(x,y) = V_1 U_1(x,y) + V_2 U_2(x,y)$$

$$\frac{\partial^2 U_i}{\partial x^2} + \frac{\partial^2 U_i}{\partial y^2} = 0,\ i = 1,2$$

$$\begin{cases} U_1 = 1 \text{ pe } \gamma_1 \text{ si } 0 \text{ pe } \gamma_2 \\ U_2 = 0 \text{ pe } \gamma_1 \text{ si } 1 \text{ pe } \gamma_2 \end{cases}$$

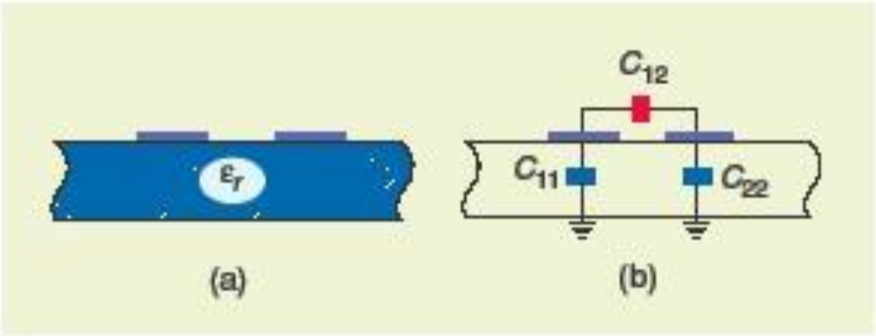
$$U_1^{'} = U_1 + U_2 \qquad U_2^{'} = U_1 - U_2$$

$$V = \frac{V_1 + V_2}{2} U_1^{'} + \frac{V_1 - V_2}{2} U_2^{'}$$

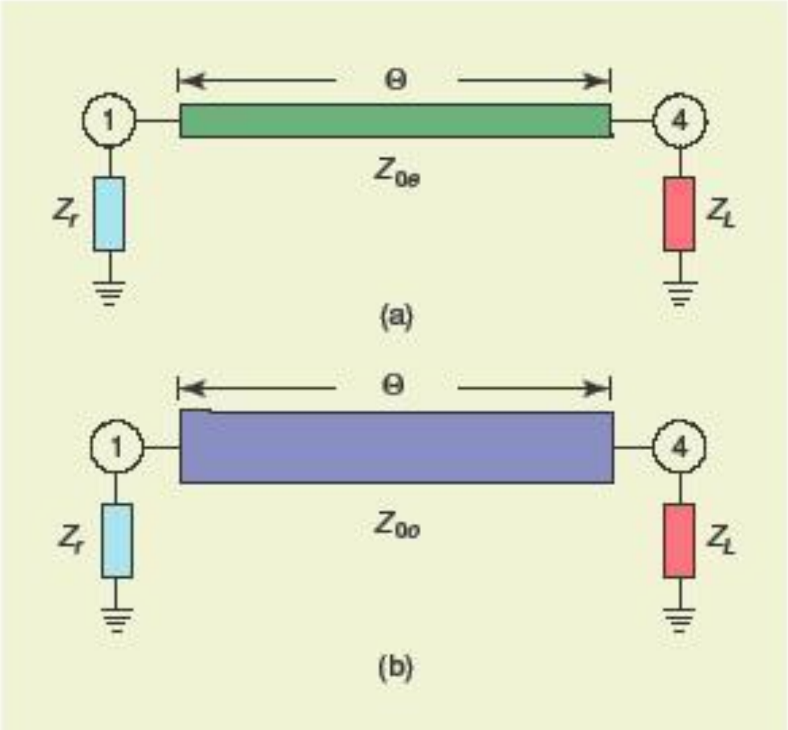
$$\frac{\partial^2 U_i^{'}}{\partial x^2} + \frac{\partial^2 U_i^{'}}{\partial y^2} = 0\ ,\ i = 1,2$$

$$\begin{cases} U_1^{'} = 1 \text{ pe } \gamma_1, \gamma_2 \\ U_2^{'} = \begin{cases} 1 \text{ pe } \gamma_1 \\ -1 \text{ pe } \gamma_2 \end{cases} \end{cases}$$

Exemplu

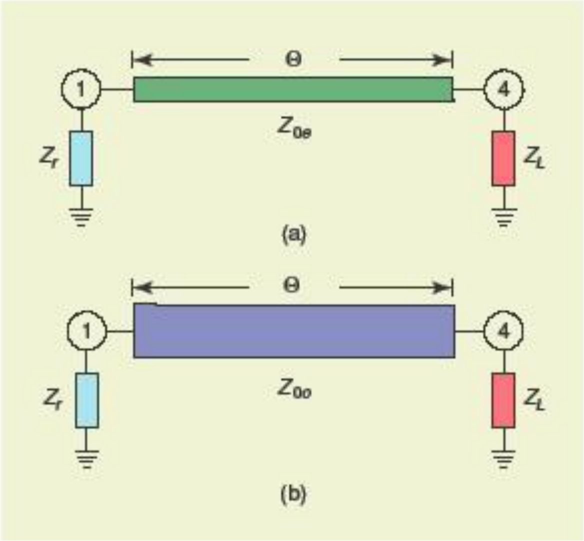


$$Z_{0e} = \frac{1}{vC_e}$$



$$Z_{0o} = \frac{1}{vC_o}$$

Calculul pe modul par si impar



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} \cos\theta & jZ_{0e}\sin\theta \\ j\dfrac{\sin\theta}{Z_{0e}} & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} \cos\theta & jZ_{0o}\sin\theta \\ j\dfrac{\sin\theta}{Z_{0o}} & \cos\theta \end{bmatrix}$$

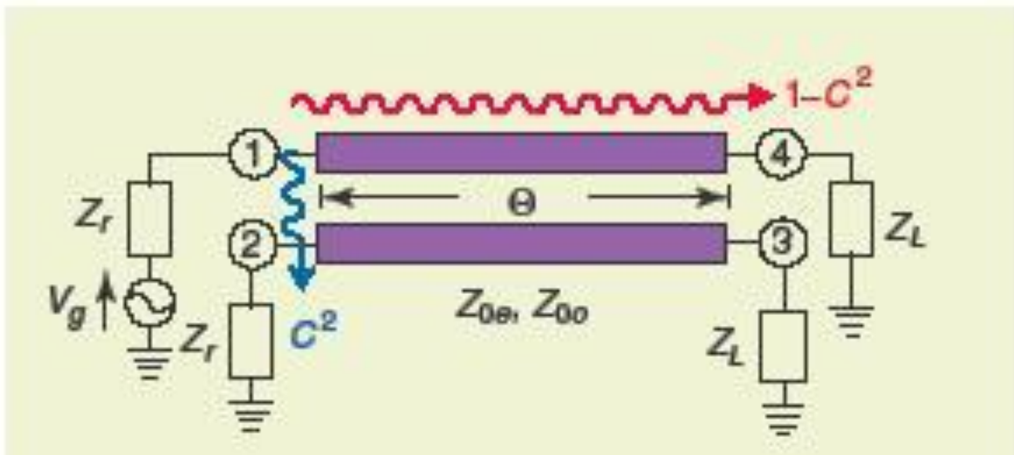
$$\Gamma_e = \frac{(Z_L - Z_r)\cos\theta + j\left(Z_{0e} - \frac{1}{Z_{0e}} Z_L Z_r\right)\sin\theta}{(Z_L + Z_r)\cos\theta + j\left(Z_{0e} + \frac{1}{Z_{0e}} Z_L Z_r\right)\sin\theta}$$

$$T_e = \frac{2}{\left(\sqrt{\frac{Z_L}{Z_r}} + \sqrt{\frac{Z_r}{Z_L}}\right)\cos\theta + j\left(\frac{Z_{0e}}{\sqrt{Z_L Z_r}} + \frac{1}{Z_{0e}}\sqrt{Z_L Z_r}\right)\sin\theta}$$

$$\Gamma_o = \frac{(Z_L - Z_r)\cos\theta + j\left(Z_{0o} - \frac{1}{Z_{0o}} Z_L Z_r\right)\sin\theta}{(Z_L + Z_r)\cos\theta + j\left(Z_{0o} + \frac{1}{Z_{0o}} Z_L Z_r\right)\sin\theta}$$

$$T_e = \frac{2}{\left(\sqrt{\frac{Z_L}{Z_r}} + \sqrt{\frac{Z_r}{Z_L}}\right)\cos\theta + j\left(\frac{Z_{0o}}{\sqrt{Z_L Z_r}} + \frac{1}{Z_{0o}}\sqrt{Z_L Z_r}\right)\sin\theta}$$

Parametrii S



$$S_{11} = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

$$S_{21} = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

$$S_{31} = \frac{1}{2}T_e - \frac{1}{2}T_o$$

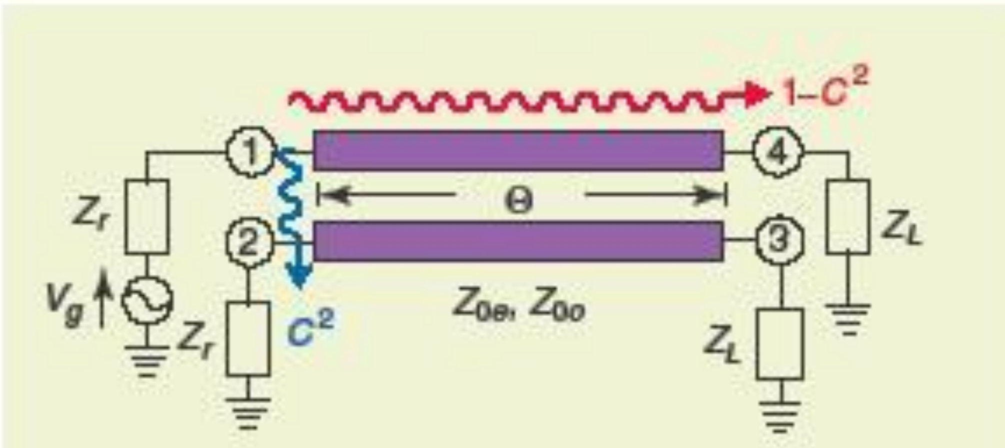
$$S_{41} = \frac{1}{2}T_e + \frac{1}{2}T_o$$

$$S_{11} = \frac{1}{2}\left(\Gamma_e + \Gamma_o\right) = \frac{\Re e(N) + j\Im m(N)}{D}$$

$$\Re e(N) = \left(Z_L^2 - Z_r^2\right)cos^2\theta - \left[\left(Z_{0e}Z_{0o} - \frac{1}{Z_{0e}Z_{0o}}\left(Z_L Z_r\right)^2\right)\right]sin^2\theta$$

$$\Im m(N) = \left[\left(Z_{0e} + Z_{0o}\right)Z_L - Z_L Z_r\left(\frac{1}{Z_{0e}} + \frac{1}{Z_{0o}}\right)Z_r\right]sin\theta\, cos\theta$$

Parametrii Z



$$Z_c^+ = \frac{Z_{ce} + Z_{co}}{2}$$

$$Z_c^- = \frac{Z_{ce} - Z_{co}}{2}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_c^+ \coth(\gamma z) & Z_c^- \coth(\gamma z) & \dfrac{Z_c^-}{\mathrm{sh}(\gamma z)} & \dfrac{Z_c^+}{\mathrm{sh}(\gamma z)} \\ Z_c^- \coth(\gamma z) & Z_c^+ \coth(\gamma z) & \dfrac{Z_c^+}{\mathrm{sh}(\gamma z)} & \dfrac{Z_c^-}{\mathrm{sh}(\gamma z)} \\ \dfrac{Z_c^-}{\mathrm{sh}(\gamma z)} & \dfrac{Z_c^+}{\mathrm{sh}(\gamma z)} & Z_c^+ \coth(\gamma z) & Z_c^- \coth(\gamma z) \\ \dfrac{Z_c^+}{\mathrm{sh}(\gamma z)} & \dfrac{Z_c^-}{\mathrm{sh}(\gamma z)} & Z_c^- \coth(\gamma z) & Z_c^+ \coth(\gamma z) \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$