

## Subject no. 1

1.  $y = Y/Y_0 = Z_0/Z = 75\Omega / (35.1 + j \cdot 59.1)\Omega = 0.557 - j \cdot 0.938$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0344 + j \cdot 0.0215)] / (0.02 + 0.0344 + j \cdot 0.0215)$   
 $\Gamma = (-0.364) + j \cdot (-0.251) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.442 \angle -145.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 23.50\text{dB}$   
 $P_{\text{in}} = 3.25\text{mW} = 5.119\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.119\text{dBm} - 23.50\text{dB} = -18.38\text{dBm} = 14.517\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.613$ ,  $Z_{\text{CE}} = 102.09\Omega$ ,  $Z_{\text{CO}} = 24.49\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 57)\Omega} = 53.39\Omega$   
b)  $Z_L = 57\Omega$  series with  $0.37\text{pF}$  capacitor at  $7.0\text{GHz} = 57.00\Omega + j \cdot (-61.45)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 23.12\Omega + j \cdot (24.93)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 15.30\text{dB}$ ):  $G = G_1 + G_4 = 6.7 + 10.9 = 17.6\text{dB}$ ;  $G = G_2 + G_3 = 7.8 + 8.3 = 16.1\text{dB}$ ;  $G = G_2 + G_4 = 7.8 + 10.9 = 18.7\text{dB}$ ;  $G = G_3 + G_4 = 8.3 + 10.9 = 19.2\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.55\text{dB} = 1.135$ ,  $F_2 = 0.87\text{dB} = 1.222$ ,  $F_3 = 1.06\text{dB} = 1.276$ ,  $F_4 = 1.28\text{dB} = 1.343$ ,  $G_1 = 6.7\text{dB} = 4.677$ ,  
 $G_2 = 7.8\text{dB} = 6.026$ ;  $F(1,4) = 1.135 + (1.343 - 1)/4.677 = 1.208 = 0.82\text{dB}$ ;  $F(2,3) = 1.222 + (1.276 - 1)/6.026 = 1.279 = 1.07\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.662 < 1$  ;  $|S_{22}| = 0.516 < 1$  ;  $K = 1.066 > 1$  ;  $|\Delta| = |(-0.198) + j \cdot (0.223)| = 0.298 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 25.61 = 14.08\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.083$  ;  $C_1 = (-0.525) + j \cdot (-0.116)$  ;  $\Gamma_S = (-0.862) + j \cdot (0.191) = 0.883 \angle 167.5^\circ$   
 $B_2 = 0.739$  ;  $C_2 = (-0.171) + j \cdot (-0.321)$  ;  $\Gamma_L = (-0.392) + j \cdot (0.735) = 0.833 \angle 118.1^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 172.3^\circ$  ;  $\text{Im}(y_S) = -3.769$  ;  $\theta_{p1} = 104.9^\circ$  or  $\theta_{S2} = 20.2^\circ$  ;  $\text{Im}(y_S) = 3.769$  ;  $\theta_{p2} = 75.1^\circ$   
output:  $\theta_{L1} = 14.2^\circ$  ;  $\text{Im}(y_L) = -3.010$  ;  $\theta_{p1} = 108.4^\circ$  or  $\theta_{L2} = 47.8^\circ$  ;  $\text{Im}(y_L) = 3.010$  ;  $\theta_{p2} = 71.6^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 14.2^\circ$  ;  $\text{Im}(y_L) = -3.010 + (-3.769) = -6.779$ ;  $\theta_{p1} = 98.4^\circ$  ;  $\theta_{S1} = 172.3^\circ$  ;  
d2)  $\theta_{L2} = 47.8^\circ$  ;  $\text{Im}(y_L) = 3.010 + (-3.769) = -0.759$ ;  $\theta_{p2} = 142.8^\circ$  ;  $\theta_{S1} = 172.3^\circ$  ;  
d3)  $\theta_{L1} = 14.2^\circ$  ;  $\text{Im}(y_L) = -3.010 + (3.769) = 0.759$ ;  $\theta_{p3} = 37.2^\circ$  ;  $\theta_{S2} = 20.2^\circ$  ;  
d4)  $\theta_{L2} = 47.8^\circ$  ;  $\text{Im}(y_L) = 3.010 + (3.769) = 6.779$ ;  $\theta_{p4} = 81.6^\circ$  ;  $\theta_{S2} = 20.2^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 14.2 + 172.3 = 186.4$  ;  $\theta_p = 98.4$  ;  $A \sim 18344.4$   
e2)  $\theta_s = 47.8 + 172.3 = 220.0$  ;  $\theta_p = 142.8$  ;  $A \sim 31419.6$   
e3)  $\theta_s = 14.2 + 20.2 = 34.4$  ;  $\theta_p = 37.2$  ;  $A \sim 1279.9$   
e4)  $\theta_s = 47.8 + 20.2 = 68.0$  ;  $\theta_p = 81.6$  ;  $A \sim 5549.2$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 2

1.  $y = Y/Y_0 = Z_0/Z = 40\Omega / (54.8 + j \cdot 60.4)\Omega = 0.330 - j \cdot 0.363$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0360 - j \cdot 0.0255)] / (0.02 + 0.0360 - j \cdot 0.0255)$   
 $\Gamma = (-0.408) + j \cdot (0.269) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.489 \angle 146.6^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless ring coupler, matched at the input; Isolation  $I = 22.80\text{dB}$   
 $P_{\text{in}} = 2.70\text{mW} = 4.314\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 4.314\text{dBm} - 22.80\text{dB} = -18.49\text{dBm} = 14.170\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.531$ ,  $y_1 = 0.531$ ,  $y_2 = 0.847$ ,  $Z_1 = Z_0/y_1 = 94.2\Omega$ ,  $Z_2 = Z_0/y_2 = 59.0\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 40)\Omega} = 44.72\Omega$   
b)  $Z_L = 40\Omega$  parallel with  $0.31\text{pF}$  capacitor at  $7.4\text{GHz} = 30.02\Omega + j \cdot (-17.31)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (28.83)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 16.20\text{dB}$ ):  $G = G_1 + G_4 = 6.3 + 11.8 = 18.1\text{dB}$ ;  $G = G_2 + G_3 = 7.0 + 9.5 = 16.5\text{dB}$ ;  $G = G_2 + G_4 = 7.0 + 11.8 = 18.8\text{dB}$ ;  $G = G_3 + G_4 = 9.5 + 11.8 = 21.3\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.54\text{dB} = 1.132$ ,  $F_2 = 0.88\text{dB} = 1.225$ ,  $F_3 = 0.99\text{dB} = 1.256$ ,  $F_4 = 1.12\text{dB} = 1.294$ ,  $G_1 = 6.3\text{dB} = 4.266$ ,  $G_2 = 7.0\text{dB} = 5.012$ ;  $F(1,4) = 1.132 + (1.294 - 1)/4.266 = 1.201 = 0.80\text{dB}$ ;  $F(2,3) = 1.225 + (1.256 - 1)/5.012 = 1.283 = 1.08\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.640 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.181 > 1$  ;  $|\Delta| = |(0.241) + j \cdot (0.152)| = 0.285 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 10.67 = 10.28\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.026$  ;  $C_1 = (-0.406) + j \cdot (0.298)$  ;  $\Gamma_S = (-0.664) + j \cdot (-0.488) = 0.824 \angle -143.7^\circ$   
 $B_2 = 0.812$  ;  $C_2 = (-0.361) + j \cdot (-0.156)$  ;  $\Gamma_L = (-0.717) + j \cdot (0.310) = 0.781 \angle 156.6^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 144.6^\circ$  ;  $\text{Im}(y_S) = -2.906$  ;  $\theta_{p1} = 109.0^\circ$  or  $\theta_{S2} = 179.1^\circ$  ;  $\text{Im}(y_S) = 2.906$  ;  $\theta_{p2} = 71.0^\circ$   
output:  $\theta_{L1} = 172.4^\circ$  ;  $\text{Im}(y_L) = -2.503$  ;  $\theta_{p1} = 111.8^\circ$  or  $\theta_{L2} = 31.0^\circ$  ;  $\text{Im}(y_L) = 2.503$  ;  $\theta_{p2} = 68.2^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 172.4^\circ$  ;  $\text{Im}(y_L) = -2.503 + (-2.906) = -5.408$ ;  $\theta_{p1} = 100.5^\circ$  ;  $\theta_{S1} = 144.6^\circ$  ;  
d2)  $\theta_{L2} = 31.0^\circ$  ;  $\text{Im}(y_L) = 2.503 + (-2.906) = -0.403$ ;  $\theta_{p2} = 158.0^\circ$  ;  $\theta_{S1} = 144.6^\circ$  ;  
d3)  $\theta_{L1} = 172.4^\circ$  ;  $\text{Im}(y_L) = -2.503 + (2.906) = 0.403$ ;  $\theta_{p3} = 22.0^\circ$  ;  $\theta_{S2} = 179.1^\circ$  ;  
d4)  $\theta_{L2} = 31.0^\circ$  ;  $\text{Im}(y_L) = 2.503 + (2.906) = 5.408$ ;  $\theta_{p4} = 79.5^\circ$  ;  $\theta_{S2} = 179.1^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 172.4 + 144.6 = 317.0$  ;  $\theta_p = 100.5$  ;  $A \sim 31846.3$   
e2)  $\theta_s = 31.0 + 144.6 = 175.6$  ;  $\theta_p = 158.0$  ;  $A \sim 27749.5$   
e3)  $\theta_s = 172.4 + 179.1 = 351.5$  ;  $\theta_p = 22.0$  ;  $A \sim 7719.3$   
e4)  $\theta_s = 31.0 + 179.1 = 210.1$  ;  $\theta_p = 79.5$  ;  $A \sim 16710.1$   
Smallest substrate area is occupied by solution e3 (d3)

### Subject no. 3

1.  $y = Y/Y_0 = Z_0/Z = 80\Omega / (39.8 - j \cdot 57.3)\Omega = 0.654 + j \cdot 0.942$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0196 - j \cdot 0.0397)] / (0.02 + 0.0196 - j \cdot 0.0397)$   
 $\Gamma = (-0.496) + j \cdot (0.505) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.708 \angle 134.5^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless ring coupler, matched at the input; Isolation  $I = 24.00\text{dB}$   
 $P_{\text{in}} = 2.30\text{mW} = 3.617\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 3.617\text{dBm} - 24.00\text{dB} = -20.38\text{dBm} = 9.156\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.579$ ,  $y_1 = 0.579$ ,  $y_2 = 0.815$ ,  $Z_1 = Z_0/y_1 = 86.4\Omega$ ,  $Z_2 = Z_0/y_2 = 61.3\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 57)\Omega} = 53.39\Omega$   
b)  $Z_L = 57\Omega$  series with  $0.34\text{pF}$  capacitor at  $8.3\text{GHz} = 57.00\Omega + j \cdot (-56.40)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 25.27\Omega + j \cdot (25.00)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 14.05\text{dB}$ ):  $G = G_1 + G_4 = 5.1 + 11.8 = 16.9\text{dB}$ ;  $G = G_2 + G_3 = 8.8 + 8.5 = 17.3\text{dB}$ ;  $G = G_2 + G_4 = 8.8 + 11.8 = 20.6\text{dB}$ ;  $G = G_3 + G_4 = 8.5 + 11.8 = 20.3\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.50\text{dB} = 1.122$ ,  $F_2 = 0.84\text{dB} = 1.213$ ,  $F_3 = 0.99\text{dB} = 1.256$ ,  $F_4 = 1.15\text{dB} = 1.303$ ,  $G_1 = 5.1\text{dB} = 3.236$ ,  
 $G_2 = 8.8\text{dB} = 7.586$ ;  $F(1,4) = 1.122 + (1.303 - 1)/3.236 = 1.216 = 0.85\text{dB}$ ;  $F(2,3) = 1.213 + (1.256 - 1)/7.586 = 1.253 = 0.98\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.609 < 1$  ;  $|S_{22}| = 0.557 < 1$  ;  $K = 1.203 > 1$  ;  $|\Delta| = |(0.236) + j \cdot (-0.069)| = 0.246 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 8.95 = 9.52\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.000$  ;  $C_1 = (-0.220) + j \cdot (0.437)$  ;  $\Gamma_S = (-0.363) + j \cdot (-0.721) = 0.807 \angle -116.7^\circ$   
 $B_2 = 0.879$  ;  $C_2 = (-0.424) + j \cdot (-0.045)$  ;  $\Gamma_L = (-0.779) + j \cdot (0.082) = 0.783 \angle 174.0^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 130.3^\circ$  ;  $\text{Im}(y_S) = -2.736$  ;  $\theta_{p1} = 110.1^\circ$  or  $\theta_{S2} = 166.4^\circ$  ;  $\text{Im}(y_S) = 2.736$  ;  $\theta_{p2} = 69.9^\circ$   
output:  $\theta_{L1} = 163.8^\circ$  ;  $\text{Im}(y_L) = -2.517$  ;  $\theta_{p1} = 111.7^\circ$  or  $\theta_{L2} = 22.3^\circ$  ;  $\text{Im}(y_L) = 2.517$  ;  $\theta_{p2} = 68.3^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 163.8^\circ$  ;  $\text{Im}(y_L) = -2.517 + (-2.736) = -5.252$ ;  $\theta_{p1} = 100.8^\circ$  ;  $\theta_{S1} = 130.3^\circ$  ;  
d2)  $\theta_{L2} = 22.3^\circ$  ;  $\text{Im}(y_L) = 2.517 + (-2.736) = -0.219$ ;  $\theta_{p2} = 167.7^\circ$  ;  $\theta_{S1} = 130.3^\circ$  ;  
d3)  $\theta_{L1} = 163.8^\circ$  ;  $\text{Im}(y_L) = -2.517 + (2.736) = 0.219$ ;  $\theta_{p3} = 12.3^\circ$  ;  $\theta_{S2} = 166.4^\circ$  ;  
d4)  $\theta_{L2} = 22.3^\circ$  ;  $\text{Im}(y_L) = 2.517 + (2.736) = 5.252$ ;  $\theta_{p4} = 79.2^\circ$  ;  $\theta_{S2} = 166.4^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 163.8 + 130.3 = 294.0$  ;  $\theta_p = 100.8$  ;  $A \sim 29634.0$   
e2)  $\theta_s = 22.3 + 130.3 = 152.5$  ;  $\theta_p = 167.7$  ;  $A \sim 25570.5$   
e3)  $\theta_s = 163.8 + 166.4 = 330.2$  ;  $\theta_p = 12.3$  ;  $A \sim 4077.5$   
e4)  $\theta_s = 22.3 + 166.4 = 188.7$  ;  $\theta_p = 79.2$  ;  $A \sim 14948.2$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 4

1.  $y = Y/Y_0 = Z_0/Z = 35\Omega / (37.3 + j \cdot 59.1)\Omega = 0.267 - j \cdot 0.424$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0342 + j \cdot 0.0247)] / (0.02 + 0.0342 + j \cdot 0.0247)$   
 $\Gamma = (-0.389) + j \cdot (-0.278) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.478 \angle -144.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless ring coupler, matched at the input; Isolation  $I = 21.60\text{dB}$

$P_{\text{in}} = 3.25\text{mW} = 5.119\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.119\text{dBm} - 21.60\text{dB} = -16.48\text{dBm} = 22.485\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.513$ ,  $y_1 = 0.513$ ,  $y_2 = 0.858$ ,  $Z_1 = Z_0/y_1 = 97.5\Omega$ ,  $Z_2 = Z_0/y_2 = 58.2\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 27)\Omega} = 36.74\Omega$

b)  $Z_L = 27\Omega$  series with  $1.24\text{nH}$  inductor at  $8.0\text{GHz} = 27.00\Omega + j \cdot (62.33)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 7.90\Omega + j \cdot (-18.24)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.30\text{dB}$ ):  $G = G_1 + G_4 = 6.4 + 11.6 = 18.0\text{dB}$ ;  $G = G_2 + G_3 = 8.1 + 8.8 = 16.9\text{dB}$ ;  $G = G_2 + G_4 = 8.1 + 11.6 = 19.7\text{dB}$ ;  $G = G_3 + G_4 = 8.8 + 11.6 = 20.4\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.64\text{dB} = 1.159$ ,  $F_2 = 0.83\text{dB} = 1.211$ ,  $F_3 = 0.98\text{dB} = 1.253$ ,  $F_4 = 1.27\text{dB} = 1.340$ ,  $G_1 = 6.4\text{dB} = 4.365$ ,  $G_2 = 8.1\text{dB} = 6.457$ ;  $F(1,4) = 1.159 + (1.340 - 1)/4.365 = 1.237 = 0.92\text{dB}$ ;  $F(2,3) = 1.211 + (1.253 - 1)/6.457 = 1.263 = 1.01\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.640 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.184 > 1$  ;  $|\Delta| = |(0.251) + j \cdot (0.130)| = 0.283 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 10.55 = 10.23\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.027$  ;  $C_1 = (-0.393) + j \cdot (0.315)$  ;  $\Gamma_S = (-0.642) + j \cdot (-0.515) = 0.824 \angle -141.3^\circ$

$B_2 = 0.813$  ;  $C_2 = (-0.367) + j \cdot (-0.145)$  ;  $\Gamma_L = (-0.726) + j \cdot (0.288) = 0.781 \angle 158.4^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 143.4^\circ$  ;  $\text{Im}(y_S) = -2.903$  ;  $\theta_{p1} = 109.0^\circ$  or  $\theta_{S2} = 177.9^\circ$  ;  $\text{Im}(y_S) = 2.903$  ;  $\theta_{p2} = 71.0^\circ$

output:  $\theta_{L1} = 171.5^\circ$  ;  $\text{Im}(y_L) = -2.501$  ;  $\theta_{p1} = 111.8^\circ$  or  $\theta_{L2} = 30.1^\circ$  ;  $\text{Im}(y_L) = 2.501$  ;  $\theta_{p2} = 68.2^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 171.5^\circ$  ;  $\text{Im}(y_L) = -2.501 + (-2.903) = -5.404$ ;  $\theta_{p1} = 100.5^\circ$  ;  $\theta_{S1} = 143.4^\circ$  ;

d2)  $\theta_{L2} = 30.1^\circ$  ;  $\text{Im}(y_L) = 2.501 + (-2.903) = -0.402$ ;  $\theta_{p2} = 158.1^\circ$  ;  $\theta_{S1} = 143.4^\circ$  ;

d3)  $\theta_{L1} = 171.5^\circ$  ;  $\text{Im}(y_L) = -2.501 + (2.903) = 0.402$ ;  $\theta_{p3} = 21.9^\circ$  ;  $\theta_{S2} = 177.9^\circ$  ;

d4)  $\theta_{L2} = 30.1^\circ$  ;  $\text{Im}(y_L) = 2.501 + (2.903) = 5.404$ ;  $\theta_{p4} = 79.5^\circ$  ;  $\theta_{S2} = 177.9^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 171.5 + 143.4 = 314.8$  ;  $\theta_p = 100.5$  ;  $A \sim 31636.5$

e2)  $\theta_s = 30.1 + 143.4 = 173.5$  ;  $\theta_p = 158.1$  ;  $A \sim 27425.4$

e3)  $\theta_s = 171.5 + 177.9 = 349.4$  ;  $\theta_p = 21.9$  ;  $A \sim 7659.8$

e4)  $\theta_s = 30.1 + 177.9 = 208.1$  ;  $\theta_p = 79.5$  ;  $A \sim 16543.8$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 5

1.  $y = Y/Y_0 = Z_0/Z = 85\Omega / (47.4 - j \cdot 34.1)\Omega = 1.182 + j \cdot 0.850$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0142 + j \cdot 0.0265)] / (0.02 + 0.0142 + j \cdot 0.0265)$   
 $\Gamma = (-0.269) + j \cdot (-0.566) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.627 \angle -115.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 20.00\text{dB}$

$P_{\text{in}} = 2.30\text{mW} = 3.617\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 3.617\text{dBm} - 20.00\text{dB} = -16.38\text{dBm} = 23.000\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.468$ ,  $Z_{\text{CE}} = 83.03\Omega$ ,  $Z_{\text{CO}} = 30.11\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 31)\Omega} = 39.37\Omega$

b)  $Z_L = 31\Omega$  series with  $0.41\text{pF}$  capacitor at  $8.5\text{GHz} = 31.00\Omega + j \cdot (-45.67)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 15.77\Omega + j \cdot (23.23)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.75\text{dB}$ ):  $G = G_1 + G_4 = 6.7 + 10.4 = 17.1\text{dB}$ ;  $G = G_2 + G_3 = 7.4 + 9.5 = 16.9\text{dB}$ ;  $G = G_2 + G_4 = 7.4 + 10.4 = 17.8\text{dB}$ ;  $G = G_3 + G_4 = 9.5 + 10.4 = 19.9\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.59\text{dB} = 1.146$ ,  $F_2 = 0.70\text{dB} = 1.175$ ,  $F_3 = 1.03\text{dB} = 1.268$ ,  $F_4 = 1.19\text{dB} = 1.315$ ,  $G_1 = 6.7\text{dB} = 4.677$ ,  $G_2 = 7.4\text{dB} = 5.495$ ;  $F(1,4) = 1.146 + (1.315 - 1)/4.677 = 1.213 = 0.84\text{dB}$ ;  $F(2,3) = 1.175 + (1.268 - 1)/5.495 = 1.232 = 0.91\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.634 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.213 > 1$  ;  $|\Delta| = |(0.264) + j \cdot (0.026)| = 0.265 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 9.79 = 9.91\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.029$  ;  $C_1 = (-0.327) + j \cdot (0.383)$  ;  $\Gamma_S = (-0.530) + j \cdot (-0.620) = 0.815 \angle -130.5^\circ$

$B_2 = 0.830$  ;  $C_2 = (-0.389) + j \cdot (-0.103)$  ;  $\Gamma_L = (-0.749) + j \cdot (0.199) = 0.775 \angle 165.1^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 137.6^\circ$  ;  $\text{Im}(y_S) = -2.818$  ;  $\theta_{p1} = 109.5^\circ$  or  $\theta_{S2} = 172.9^\circ$  ;  $\text{Im}(y_S) = 2.818$  ;  $\theta_{p2} = 70.5^\circ$

output:  $\theta_{L1} = 167.8^\circ$  ;  $\text{Im}(y_L) = -2.453$  ;  $\theta_{p1} = 112.2^\circ$  or  $\theta_{L2} = 27.0^\circ$  ;  $\text{Im}(y_L) = 2.453$  ;  $\theta_{p2} = 67.8^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 167.8^\circ$  ;  $\text{Im}(y_L) = -2.453 + (-2.818) = -5.271$ ;  $\theta_{p1} = 100.7^\circ$  ;  $\theta_{S1} = 137.6^\circ$  ;

d2)  $\theta_{L2} = 27.0^\circ$  ;  $\text{Im}(y_L) = 2.453 + (-2.818) = -0.364$ ;  $\theta_{p2} = 160.0^\circ$  ;  $\theta_{S1} = 137.6^\circ$  ;

d3)  $\theta_{L1} = 167.8^\circ$  ;  $\text{Im}(y_L) = -2.453 + (2.818) = 0.364$ ;  $\theta_{p3} = 20.0^\circ$  ;  $\theta_{S2} = 172.9^\circ$  ;

d4)  $\theta_{L2} = 27.0^\circ$  ;  $\text{Im}(y_L) = 2.453 + (2.818) = 5.271$ ;  $\theta_{p4} = 79.3^\circ$  ;  $\theta_{S2} = 172.9^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 167.8 + 137.6 = 305.4$  ;  $\theta_p = 100.7$  ;  $A \sim 30768.3$

e2)  $\theta_s = 27.0 + 137.6 = 164.6$  ;  $\theta_p = 160.0$  ;  $A \sim 26332.7$

e3)  $\theta_s = 167.8 + 172.9 = 340.8$  ;  $\theta_p = 20.0$  ;  $A \sim 6823.6$

e4)  $\theta_s = 27.0 + 172.9 = 200.0$  ;  $\theta_p = 79.3$  ;  $A \sim 15849.2$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 6

1.  $y = Y/Y_0 = Z_0/Z = 75\Omega / (42.8 + j \cdot 30.4)\Omega = 1.165 - j \cdot 0.827$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0200 + j \cdot 0.0115)] / (0.02 + 0.0200 + j \cdot 0.0115)$   
 $\Gamma = (-0.076) + j \cdot (-0.266) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.276 \angle -106.0^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = D + C = 29.40\text{dB}$

$P_{\text{in}} = 2.60\text{mW} = 4.150\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 4.150\text{dBm} - 29.40\text{dB} = -25.25\text{dBm} = 2.985\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.462$ ,  $Z_{\text{CE}} = 82.46\Omega$ ,  $Z_{\text{CO}} = 30.32\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 27)\Omega} = 36.74\Omega$

b)  $Z_L = 27\Omega$  parallel with  $0.48\text{pF}$  capacitor at  $6.5\text{GHz} = 21.09\Omega + j \cdot (-11.16)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (26.46)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.35\text{dB}$ ):  $G = G_1 + G_4 = 6.2 + 10.7 = 16.9\text{dB}$ ;  $G = G_2 + G_3 = 8.6 + 8.5 = 17.1\text{dB}$ ;  $G = G_2 + G_4 = 8.6 + 10.7 = 19.3\text{dB}$ ;  $G = G_3 + G_4 = 8.5 + 10.7 = 19.2\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.68\text{dB} = 1.169$ ,  $F_2 = 0.84\text{dB} = 1.213$ ,  $F_3 = 1.09\text{dB} = 1.285$ ,  $F_4 = 1.21\text{dB} = 1.321$ ,  $G_1 = 6.2\text{dB} = 4.169$ ,  $G_2 = 8.6\text{dB} = 7.244$ ;  $F(1,4) = 1.169 + (1.321 - 1)/4.169 = 1.247 = 0.96\text{dB}$ ;  $F(2,3) = 1.213 + (1.285 - 1)/7.244 = 1.258 = 1.00\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.627 < 1$  ;  $|S_{22}| = 0.551 < 1$  ;  $K = 1.227 > 1$  ;  $|\Delta| = |(0.253) + j \cdot (-0.026)| = 0.255 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 9.34 = 9.70\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.025$  ;  $C_1 = (-0.284) + j \cdot (0.413)$  ;  $\Gamma_S = (-0.458) + j \cdot (-0.667) = 0.809 \angle -124.5^\circ$

$B_2 = 0.846$  ;  $C_2 = (-0.401) + j \cdot (-0.080)$  ;  $\Gamma_L = (-0.758) + j \cdot (0.151) = 0.773 \angle 168.7^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 134.3^\circ$  ;  $\text{Im}(y_S) = -2.757$  ;  $\theta_{p1} = 109.9^\circ$  or  $\theta_{S2} = 170.2^\circ$  ;  $\text{Im}(y_S) = 2.757$  ;  $\theta_{p2} = 70.1^\circ$

output:  $\theta_{L1} = 165.9^\circ$  ;  $\text{Im}(y_L) = -2.434$  ;  $\theta_{p1} = 112.3^\circ$  or  $\theta_{L2} = 25.3^\circ$  ;  $\text{Im}(y_L) = 2.434$  ;  $\theta_{p2} = 67.7^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 165.9^\circ$  ;  $\text{Im}(y_L) = -2.434 + (-2.757) = -5.192$ ;  $\theta_{p1} = 100.9^\circ$  ;  $\theta_{S1} = 134.3^\circ$  ;

d2)  $\theta_{L2} = 25.3^\circ$  ;  $\text{Im}(y_L) = 2.434 + (-2.757) = -0.323$ ;  $\theta_{p2} = 162.1^\circ$  ;  $\theta_{S1} = 134.3^\circ$  ;

d3)  $\theta_{L1} = 165.9^\circ$  ;  $\text{Im}(y_L) = -2.434 + (2.757) = 0.323$ ;  $\theta_{p3} = 17.9^\circ$  ;  $\theta_{S2} = 170.2^\circ$  ;

d4)  $\theta_{L2} = 25.3^\circ$  ;  $\text{Im}(y_L) = 2.434 + (2.757) = 5.192$ ;  $\theta_{p4} = 79.1^\circ$  ;  $\theta_{S2} = 170.2^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 165.9 + 134.3 = 300.2$  ;  $\theta_p = 100.9$  ;  $A \sim 30288.8$

e2)  $\theta_s = 25.3 + 134.3 = 159.6$  ;  $\theta_p = 162.1$  ;  $A \sim 25866.9$

e3)  $\theta_s = 165.9 + 170.2 = 336.1$  ;  $\theta_p = 17.9$  ;  $A \sim 6021.0$

e4)  $\theta_s = 25.3 + 170.2 = 195.5$  ;  $\theta_p = 79.1$  ;  $A \sim 15466.7$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 7

1.  $y = Y/Y_0 = Z_0/Z = 30\Omega / (51.9 - j \cdot 49.5)\Omega = 0.303 + j \cdot 0.289$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0327 + j \cdot 0.0259)] / (0.02 + 0.0327 + j \cdot 0.0259)$   
 $\Gamma = (-0.389) + j \cdot (-0.300) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.491 \angle -142.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless ring coupler, matched at the input; Isolation  $I = 24.00\text{dB}$

$P_{in} = 1.30\text{mW} = 1.139\text{dBm}$ ;  $P_{is} = P_{in} - I = 1.139\text{dBm} - 24.00\text{dB} = -22.86\text{dBm} = 5.175\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.562$ ,  $y_1 = 0.562$ ,  $y_2 = 0.827$ ,  $Z_1 = Z_0/y_1 = 88.9\Omega$ ,  $Z_2 = Z_0/y_2 = 60.5\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 31)\Omega} = 39.37\Omega$

b)  $Z_L = 31\Omega$  parallel with  $0.26\text{pF}$  capacitor at  $9.3\text{GHz} = 25.37\Omega + j \cdot (-11.95)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{in} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (23.55)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.65\text{dB}$ ):  $G = G_1 + G_4 = 5.7 + 10.6 = 16.3\text{dB}$ ;  $G = G_2 + G_3 = 7.3 + 9.8 = 17.1\text{dB}$ ;  $G = G_2 + G_4 = 7.3 + 10.6 = 17.9\text{dB}$ ;  $G = G_3 + G_4 = 9.8 + 10.6 = 20.4\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.53\text{dB} = 1.130$ ,  $F_2 = 0.71\text{dB} = 1.178$ ,  $F_3 = 0.97\text{dB} = 1.250$ ,  $F_4 = 1.29\text{dB} = 1.346$ ,  $G_1 = 5.7\text{dB} = 3.715$ ,  $G_2 = 7.3\text{dB} = 5.370$ ;  $F(1,4) = 1.130 + (1.346 - 1)/3.715 = 1.223 = 0.87\text{dB}$ ;  $F(2,3) = 1.178 + (1.250 - 1)/5.370 = 1.242 = 0.94\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.653 < 1$  ;  $|S_{22}| = 0.519 < 1$  ;  $K = 1.090 > 1$  ;  $|\Delta| = |(-0.149) + j \cdot (0.243)| = 0.285 < 1$

b\_1)  $G_{Tmax} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 23.66 = 13.74\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.076$  ;  $C_1 = (-0.526) + j \cdot (-0.083)$  ;  $\Gamma_S = (-0.856) + j \cdot (0.135) = 0.866 \angle 171.0^\circ$

$B_2 = 0.762$  ;  $C_2 = (-0.199) + j \cdot (-0.316)$  ;  $\Gamma_L = (-0.434) + j \cdot (0.690) = 0.815 \angle 122.2^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 169.5^\circ$  ;  $\text{Im}(y_S) = -3.465$  ;  $\theta_{p1} = 106.1^\circ$  or  $\theta_{S2} = 19.5^\circ$  ;  $\text{Im}(y_S) = 3.465$  ;  $\theta_{p2} = 73.9^\circ$

output:  $\theta_{L1} = 11.2^\circ$  ;  $\text{Im}(y_L) = -2.814$  ;  $\theta_{p1} = 109.6^\circ$  or  $\theta_{L2} = 46.6^\circ$  ;  $\text{Im}(y_L) = 2.814$  ;  $\theta_{p2} = 70.4^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 11.2^\circ$  ;  $\text{Im}(y_L) = -2.814 + (-3.465) = -6.279$ ;  $\theta_{p1} = 99.0^\circ$  ;  $\theta_{S1} = 169.5^\circ$  ;

d2)  $\theta_{L2} = 46.6^\circ$  ;  $\text{Im}(y_L) = 2.814 + (-3.465) = -0.651$ ;  $\theta_{p2} = 146.9^\circ$  ;  $\theta_{S1} = 169.5^\circ$  ;

d3)  $\theta_{L1} = 11.2^\circ$  ;  $\text{Im}(y_L) = -2.814 + (3.465) = 0.651$ ;  $\theta_{p3} = 33.1^\circ$  ;  $\theta_{S2} = 19.5^\circ$  ;

d4)  $\theta_{L2} = 46.6^\circ$  ;  $\text{Im}(y_L) = 2.814 + (3.465) = 6.279$ ;  $\theta_{p4} = 81.0^\circ$  ;  $\theta_{S2} = 19.5^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 11.2 + 169.5 = 180.7$  ;  $\theta_p = 99.0$  ;  $A \sim 17898.3$

e2)  $\theta_s = 46.6 + 169.5 = 216.1$  ;  $\theta_p = 146.9$  ;  $A \sim 31751.0$

e3)  $\theta_s = 11.2 + 19.5 = 30.7$  ;  $\theta_p = 33.1$  ;  $A \sim 1015.2$

e4)  $\theta_s = 46.6 + 19.5 = 66.1$  ;  $\theta_p = 81.0$  ;  $A \sim 5350.6$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 8

1.  $y = Y/Y_0 = Z_0/Z = 80\Omega / (62.2 - j \cdot 51.2)\Omega = 0.767 + j \cdot 0.631$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0325 - j \cdot 0.0195)] / (0.02 + 0.0325 - j \cdot 0.0195)$   
 $\Gamma = (-0.330) + j \cdot (0.249) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.414 \angle 143.0^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = D + C = 28.25\text{dB}$

$P_{\text{in}} = 2.15\text{mW} = 3.324\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 3.324\text{dBm} - 28.25\text{dB} = -24.93\text{dBm} = 3.217\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.449$ ,  $Z_{\text{CE}} = 81.11\Omega$ ,  $Z_{\text{CO}} = 30.82\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 41)\Omega} = 45.28\Omega$

b)  $Z_L = 41\Omega$  series with  $0.88\text{nH}$  inductor at  $8.1\text{GHz} = 41.00\Omega + j \cdot (44.79)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 22.80\Omega + j \cdot (-24.90)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.10\text{dB}$ ):  $G = G_1 + G_4 = 6.5 + 10.2 = 16.7\text{dB}$ ;  $G = G_2 + G_3 = 7.5 + 9.1 = 16.6\text{dB}$ ;  $G = G_2 + G_4 = 7.5 + 10.2 = 17.7\text{dB}$ ;  $G = G_3 + G_4 = 9.1 + 10.2 = 19.3\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.51\text{dB} = 1.125$ ,  $F_2 = 0.83\text{dB} = 1.211$ ,  $F_3 = 1.03\text{dB} = 1.268$ ,  $F_4 = 1.17\text{dB} = 1.309$ ,  $G_1 = 6.5\text{dB} = 4.467$ ,  $G_2 = 7.5\text{dB} = 5.623$ ;  $F(1,4) = 1.125 + (1.309 - 1)/4.467 = 1.194 = 0.77\text{dB}$ ;  $F(2,3) = 1.211 + (1.268 - 1)/5.623 = 1.266 = 1.02\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.605 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.169 > 1$  ;  $|\Delta| = |(-0.037) + j \cdot (0.262)| = 0.264 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 16.34 = 12.13\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.026$  ;  $C_1 = (-0.486) + j \cdot (0.119)$  ;  $\Gamma_S = (-0.778) + j \cdot (-0.191) = 0.801 \angle -166.2^\circ$

$B_2 = 0.835$  ;  $C_2 = (-0.242) + j \cdot (-0.321)$  ;  $\Gamma_L = (-0.457) + j \cdot (0.606) = 0.759 \angle 127.0^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 154.7^\circ$  ;  $\text{Im}(y_S) = -2.674$  ;  $\theta_{p1} = 110.5^\circ$  or  $\theta_{S2} = 11.5^\circ$  ;  $\text{Im}(y_S) = 2.674$  ;  $\theta_{p2} = 69.5^\circ$

output:  $\theta_{L1} = 6.2^\circ$  ;  $\text{Im}(y_L) = -2.333$  ;  $\theta_{p1} = 113.2^\circ$  or  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.333$  ;  $\theta_{p2} = 66.8^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 6.2^\circ$  ;  $\text{Im}(y_L) = -2.333 + (-2.674) = -5.007$ ;  $\theta_{p1} = 101.3^\circ$  ;  $\theta_{S1} = 154.7^\circ$  ;

d2)  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.333 + (-2.674) = -0.341$ ;  $\theta_{p2} = 161.2^\circ$  ;  $\theta_{S1} = 154.7^\circ$  ;

d3)  $\theta_{L1} = 6.2^\circ$  ;  $\text{Im}(y_L) = -2.333 + (2.674) = 0.341$ ;  $\theta_{p3} = 18.8^\circ$  ;  $\theta_{S2} = 11.5^\circ$  ;

d4)  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.333 + (2.674) = 5.007$ ;  $\theta_{p4} = 78.7^\circ$  ;  $\theta_{S2} = 11.5^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 6.2 + 154.7 = 160.9$  ;  $\theta_p = 101.3$  ;  $A \sim 16296.5$

e2)  $\theta_s = 46.8 + 154.7 = 201.5$  ;  $\theta_p = 161.2$  ;  $A \sim 32476.7$

e3)  $\theta_s = 6.2 + 11.5 = 17.7$  ;  $\theta_p = 18.8$  ;  $A \sim 332.6$

e4)  $\theta_s = 46.8 + 11.5 = 58.3$  ;  $\theta_p = 78.7$  ;  $A \sim 4587.0$

Smallest substrate area is occupied by solution e3 (d3)



## Subject no. 9

1.  $y = Y/Y_0 = Z_0/Z = 70\Omega / (40.8 + j \cdot 68.5)\Omega = 0.449 - j \cdot 0.754$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0236 + j \cdot 0.0112)] / (0.02 + 0.0236 + j \cdot 0.0112)$   
 $\Gamma = (-0.139) + j \cdot (-0.221) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.261 \angle -122.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 22.00\text{dB}$

$P_{\text{in}} = 3.25\text{mW} = 5.119\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.119\text{dBm} - 22.00\text{dB} = -16.88\text{dBm} = 20.506\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.572$ ,  $Z_{\text{CE}} = 95.84\Omega$ ,  $Z_{\text{CO}} = 26.08\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 56)\Omega} = 52.92\Omega$

b)  $Z_L = 56\Omega$  series with  $1.05\text{nH}$  inductor at  $9.8\text{GHz} = 56.00\Omega + j \cdot (64.65)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 21.43\Omega + j \cdot (-24.74)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.35\text{dB}$ ):  $G = G_1 + G_4 = 5.4 + 10.3 = 15.7\text{dB}$ ;  $G = G_2 + G_3 = 8.1 + 9.9 = 18.0\text{dB}$ ;  $G = G_2 + G_4 = 8.1 + 10.3 = 18.4\text{dB}$ ;  $G = G_3 + G_4 = 9.9 + 10.3 = 20.2\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.64\text{dB} = 1.159$ ,  $F_2 = 0.87\text{dB} = 1.222$ ,  $F_3 = 1.01\text{dB} = 1.262$ ,  $F_4 = 1.18\text{dB} = 1.312$ ,  $G_1 = 5.4\text{dB} = 3.467$ ,  $G_2 = 8.1\text{dB} = 6.457$ ;  $F(1,4) = 1.159 + (1.312 - 1)/3.467 = 1.249 = 0.96\text{dB}$ ;  $F(2,3) = 1.222 + (1.262 - 1)/6.457 = 1.270 = 1.04\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.656 < 1$  ;  $|S_{22}| = 0.518 < 1$  ;  $K = 1.082 > 1$  ;  $|\Delta| = |(-0.166) + j \cdot (-0.237)| = 0.289 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 24.28 = 13.85\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.078$  ;  $C_1 = (-0.526) + j \cdot (-0.094)$  ;  $\Gamma_S = (-0.858) + j \cdot (0.154) = 0.872 \angle 169.9^\circ$

$B_2 = 0.754$  ;  $C_2 = (-0.189) + j \cdot (-0.318)$  ;  $\Gamma_L = (-0.420) + j \cdot (0.705) = 0.821 \angle 120.8^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 170.4^\circ$  ;  $\text{Im}(y_S) = -3.557$  ;  $\theta_{p1} = 105.7^\circ$  or  $\theta_{S2} = 19.7^\circ$  ;  $\text{Im}(y_S) = 3.557$  ;  $\theta_{p2} = 74.3^\circ$

output:  $\theta_{L1} = 12.2^\circ$  ;  $\text{Im}(y_L) = -2.872$  ;  $\theta_{p1} = 109.2^\circ$  or  $\theta_{L2} = 47.0^\circ$  ;  $\text{Im}(y_L) = 2.872$  ;  $\theta_{p2} = 70.8^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 12.2^\circ$  ;  $\text{Im}(y_L) = -2.872 + (-3.557) = -6.430$ ;  $\theta_{p1} = 98.8^\circ$  ;  $\theta_{S1} = 170.4^\circ$  ;

d2)  $\theta_{L2} = 47.0^\circ$  ;  $\text{Im}(y_L) = 2.872 + (-3.557) = -0.685$ ;  $\theta_{p2} = 145.6^\circ$  ;  $\theta_{S1} = 170.4^\circ$  ;

d3)  $\theta_{L1} = 12.2^\circ$  ;  $\text{Im}(y_L) = -2.872 + (3.557) = 0.685$ ;  $\theta_{p3} = 34.4^\circ$  ;  $\theta_{S2} = 19.7^\circ$  ;

d4)  $\theta_{L2} = 47.0^\circ$  ;  $\text{Im}(y_L) = 2.872 + (3.557) = 6.430$ ;  $\theta_{p4} = 81.2^\circ$  ;  $\theta_{S2} = 19.7^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 12.2 + 170.4 = 182.6$  ;  $\theta_p = 98.8$  ;  $A \sim 18046.2$

e2)  $\theta_s = 47.0 + 170.4 = 217.4$  ;  $\theta_p = 145.6$  ;  $A \sim 31656.0$

e3)  $\theta_s = 12.2 + 19.7 = 31.9$  ;  $\theta_p = 34.4$  ;  $A \sim 1098.5$

e4)  $\theta_s = 47.0 + 19.7 = 66.8$  ;  $\theta_p = 81.2$  ;  $A \sim 5419.4$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 10

1.  $y = Y/Y_0 = Z_0/Z = 80\Omega / (56.3 + j \cdot 57.9)\Omega = 0.691 - j \cdot 0.710$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0216 + j \cdot 0.0204)] / (0.02 + 0.0216 + j \cdot 0.0204)$   
 $\Gamma = (-0.225) + j \cdot (-0.380) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.442 \angle -120.6^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = D + C = 24.75\text{dB}$

$P_{\text{in}} = 2.10\text{mW} = 3.222\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 3.222\text{dBm} - 24.75\text{dB} = -21.53\text{dBm} = 7.034\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.627$ ,  $Z_{\text{CE}} = 104.48\Omega$ ,  $Z_{\text{CO}} = 23.93\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 74)\Omega} = 60.83\Omega$

b)  $Z_L = 74\Omega$  parallel with  $0.30\text{pF}$  capacitor at  $8.8\text{GHz} = 29.52\Omega + j \cdot (-36.24)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (61.37)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.35\text{dB}$ ):  $G = G_1 + G_4 = 6.0 + 10.8 = 16.8\text{dB}$ ;  $G = G_2 + G_3 = 8.7 + 9.3 = 18.0\text{dB}$ ;  $G = G_2 + G_4 = 8.7 + 10.8 = 19.5\text{dB}$ ;  $G = G_3 + G_4 = 9.3 + 10.8 = 20.1\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.57\text{dB} = 1.140$ ,  $F_2 = 0.77\text{dB} = 1.194$ ,  $F_3 = 0.98\text{dB} = 1.253$ ,  $F_4 = 1.24\text{dB} = 1.330$ ,  $G_1 = 6.0\text{dB} = 3.981$ ,  $G_2 = 8.7\text{dB} = 7.413$ ;  $F(1,4) = 1.140 + (1.330 - 1)/3.981 = 1.223 = 0.88\text{dB}$ ;  $F(2,3) = 1.194 + (1.253 - 1)/7.413 = 1.239 = 0.93\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.621 < 1$  ;  $|S_{22}| = 0.553 < 1$  ;  $K = 1.218 > 1$  ;  $|\Delta| = |(0.248) + j \cdot (-0.041)| = 0.252 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 9.22 = 9.65\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.017$  ;  $C_1 = (-0.262) + j \cdot (0.422)$  ;  $\Gamma_S = (-0.427) + j \cdot (-0.687) = 0.809 \angle -121.9^\circ$

$B_2 = 0.857$  ;  $C_2 = (-0.409) + j \cdot (-0.069)$  ;  $\Gamma_L = (-0.766) + j \cdot (0.128) = 0.776 \angle 170.5^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 132.9^\circ$  ;  $\text{Im}(y_S) = -2.751$  ;  $\theta_{p1} = 110.0^\circ$  or  $\theta_{S2} = 168.9^\circ$  ;  $\text{Im}(y_S) = 2.751$  ;  $\theta_{p2} = 70.0^\circ$

output:  $\theta_{L1} = 165.2^\circ$  ;  $\text{Im}(y_L) = -2.463$  ;  $\theta_{p1} = 112.1^\circ$  or  $\theta_{L2} = 24.3^\circ$  ;  $\text{Im}(y_L) = 2.463$  ;  $\theta_{p2} = 67.9^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 165.2^\circ$  ;  $\text{Im}(y_L) = -2.463 + (-2.751) = -5.214$ ;  $\theta_{p1} = 100.9^\circ$  ;  $\theta_{S1} = 132.9^\circ$  ;

d2)  $\theta_{L2} = 24.3^\circ$  ;  $\text{Im}(y_L) = 2.463 + (-2.751) = -0.288$ ;  $\theta_{p2} = 163.9^\circ$  ;  $\theta_{S1} = 132.9^\circ$  ;

d3)  $\theta_{L1} = 165.2^\circ$  ;  $\text{Im}(y_L) = -2.463 + (2.751) = 0.288$ ;  $\theta_{p3} = 16.1^\circ$  ;  $\theta_{S2} = 168.9^\circ$  ;

d4)  $\theta_{L2} = 24.3^\circ$  ;  $\text{Im}(y_L) = 2.463 + (2.751) = 5.214$ ;  $\theta_{p4} = 79.1^\circ$  ;  $\theta_{S2} = 168.9^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 165.2 + 132.9 = 298.1$  ;  $\theta_p = 100.9$  ;  $A \sim 30070.1$

e2)  $\theta_s = 24.3 + 132.9 = 157.2$  ;  $\theta_p = 163.9$  ;  $A \sim 25772.2$

e3)  $\theta_s = 165.2 + 168.9 = 334.2$  ;  $\theta_p = 16.1$  ;  $A \sim 5373.6$

e4)  $\theta_s = 24.3 + 168.9 = 193.2$  ;  $\theta_p = 79.1$  ;  $A \sim 15293.7$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 11

1.  $y = Y/Y_0 = Z_0/Z = 45\Omega / (43.8 + j \cdot 55.5)\Omega = 0.394 - j \cdot 0.500$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0297 + j \cdot 0.0108)] / (0.02 + 0.0297 + j \cdot 0.0108)$   
 $\Gamma = (-0.231) + j \cdot (-0.167) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.285 \angle -144.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless quadrature coupler, matched at the input; Isolation  $I = 22.80\text{dB}$   
 $P_{\text{in}} = 3.60\text{mW} = 5.563\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.563\text{dBm} - 22.80\text{dB} = -17.24\text{dBm} = 18.893\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.460$ ,  $y_2 = 1.126$ ,  $y_1 = 0.518$ ,  $Z_1 = Z_0/y_1 = 96.6\Omega$ ,  $Z_2 = Z_0/y_2 = 44.4\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 54)\Omega} = 51.96\Omega$   
b)  $Z_L = 54\Omega$  series with  $0.49\text{pF}$  capacitor at  $7.7\text{GHz} = 54.00\Omega + j \cdot (-42.18)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 31.05\Omega + j \cdot (24.26)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 15.05\text{dB}$ ):  $G = G_1 + G_4 = 6.6 + 11.7 = 18.3\text{dB}$ ;  $G = G_2 + G_3 = 7.5 + 8.1 = 15.6\text{dB}$ ;  $G = G_2 + G_4 = 7.5 + 11.7 = 19.2\text{dB}$ ;  $G = G_3 + G_4 = 8.1 + 11.7 = 19.8\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.50\text{dB} = 1.122$ ,  $F_2 = 0.88\text{dB} = 1.225$ ,  $F_3 = 1.01\text{dB} = 1.262$ ,  $F_4 = 1.15\text{dB} = 1.303$ ,  $G_1 = 6.6\text{dB} = 4.571$ ,  $G_2 = 7.5\text{dB} = 5.623$ ;  $F(1,4) = 1.122 + (1.303 - 1)/4.571 = 1.188 = 0.75\text{dB}$ ;  $F(2,3) = 1.225 + (1.262 - 1)/5.623 = 1.279 = 1.07\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.631 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.227 > 1$  ;  $|\Delta| = |(0.258) + j \cdot (-0.008)| = 0.258 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 9.51 = 9.78\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.029$  ;  $C_1 = (-0.303) + j \cdot (0.402)$  ;  $\Gamma_S = (-0.488) + j \cdot (-0.649) = 0.811 \angle -126.9^\circ$   
 $B_2 = 0.838$  ;  $C_2 = (-0.395) + j \cdot (-0.090)$  ;  $\Gamma_L = (-0.753) + j \cdot (0.171) = 0.772 \angle 167.2^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 135.6^\circ$  ;  $\text{Im}(y_S) = -2.776$  ;  $\theta_{p1} = 109.8^\circ$  or  $\theta_{S2} = 171.4^\circ$  ;  $\text{Im}(y_S) = 2.776$  ;  $\theta_{p2} = 70.2^\circ$   
output:  $\theta_{L1} = 166.7^\circ$  ;  $\text{Im}(y_L) = -2.430$  ;  $\theta_{p1} = 112.4^\circ$  or  $\theta_{L2} = 26.1^\circ$  ;  $\text{Im}(y_L) = 2.430$  ;  $\theta_{p2} = 67.6^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 166.7^\circ$  ;  $\text{Im}(y_L) = -2.430 + (-2.776) = -5.206$ ;  $\theta_{p1} = 100.9^\circ$  ;  $\theta_{S1} = 135.6^\circ$  ;  
d2)  $\theta_{L2} = 26.1^\circ$  ;  $\text{Im}(y_L) = 2.430 + (-2.776) = -0.347$ ;  $\theta_{p2} = 160.9^\circ$  ;  $\theta_{S1} = 135.6^\circ$  ;  
d3)  $\theta_{L1} = 166.7^\circ$  ;  $\text{Im}(y_L) = -2.430 + (2.776) = 0.347$ ;  $\theta_{p3} = 19.1^\circ$  ;  $\theta_{S2} = 171.4^\circ$  ;  
d4)  $\theta_{L2} = 26.1^\circ$  ;  $\text{Im}(y_L) = 2.430 + (2.776) = 5.206$ ;  $\theta_{p4} = 79.1^\circ$  ;  $\theta_{S2} = 171.4^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 166.7 + 135.6 = 302.3$  ;  $\theta_p = 100.9$  ;  $A \sim 30490.4$   
e2)  $\theta_s = 26.1 + 135.6 = 161.7$  ;  $\theta_p = 160.9$  ;  $A \sim 26017.6$   
e3)  $\theta_s = 166.7 + 171.4 = 338.0$  ;  $\theta_p = 19.1$  ;  $A \sim 6464.5$   
e4)  $\theta_s = 26.1 + 171.4 = 197.5$  ;  $\theta_p = 79.1$  ;  $A \sim 15626.9$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 12

1.  $y = Y/Y_0 = Z_0/Z = 70\Omega / (41.7 + j \cdot 61.5)\Omega = 0.529 - j \cdot 0.780$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0379 - j \cdot 0.0114)] / (0.02 + 0.0379 - j \cdot 0.0114)$   
 $\Gamma = (-0.335) + j \cdot (0.131) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.360 \angle 158.6^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless quadrature coupler, matched at the input; Isolation  $I = D + C = 29.95\text{dB}$   
 $P_{\text{in}} = 1.50\text{mW} = 1.761\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 1.761\text{dBm} - 29.95\text{dB} = -28.19\text{dBm} = 1.517\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.487$ ,  $y_2 = 1.145$ ,  $y_1 = 0.558$ ,  $Z_1 = Z_0/y_1 = 89.7\Omega$ ,  $Z_2 = Z_0/y_2 = 43.7\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 51)\Omega} = 50.50\Omega$   
b)  $Z_L = 51\Omega$  parallel with  $0.80\text{nH}$  inductor at  $9.2\text{GHz} = 23.01\Omega + j \cdot (25.38)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (-55.14)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 15.25\text{dB}$ ):  $G = G_1 + G_4 = 6.9 + 10.8 = 17.7\text{dB}$ ;  $G = G_2 + G_3 = 7.7 + 8.3 = 16.0\text{dB}$ ;  $G = G_2 + G_4 = 7.7 + 10.8 = 18.5\text{dB}$ ;  $G = G_3 + G_4 = 8.3 + 10.8 = 19.1\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.62\text{dB} = 1.153$ ,  $F_2 = 0.88\text{dB} = 1.225$ ,  $F_3 = 1.02\text{dB} = 1.265$ ,  $F_4 = 1.12\text{dB} = 1.294$ ,  $G_1 = 6.9\text{dB} = 4.898$ ,  
 $G_2 = 7.7\text{dB} = 5.888$ ;  $F(1,4) = 1.153 + (1.294 - 1)/4.898 = 1.214 = 0.84\text{dB}$ ;  $F(2,3) = 1.225 + (1.265 - 1)/5.888 = 1.275 = 1.05\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.606 < 1$  ;  $|S_{22}| = 0.558 < 1$  ;  $K = 1.199 > 1$  ;  $|\Delta| = |(0.232) + j \cdot (-0.076)| = 0.244 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 8.90 = 9.49\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 0.996$  ;  $C_1 = (-0.209) + j \cdot (0.440)$  ;  $\Gamma_S = (-0.346) + j \cdot (-0.729) = 0.807 \angle -115.4^\circ$   
 $B_2 = 0.884$  ;  $C_2 = (-0.428) + j \cdot (-0.039)$  ;  $\Gamma_L = (-0.782) + j \cdot (0.071) = 0.785 \angle 174.8^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 129.6^\circ$  ;  $\text{Im}(y_S) = -2.735$  ;  $\theta_{p1} = 110.1^\circ$  or  $\theta_{S2} = 165.8^\circ$  ;  $\text{Im}(y_S) = 2.735$  ;  $\theta_{p2} = 69.9^\circ$   
output:  $\theta_{L1} = 163.4^\circ$  ;  $\text{Im}(y_L) = -2.533$  ;  $\theta_{p1} = 111.5^\circ$  or  $\theta_{L2} = 21.7^\circ$  ;  $\text{Im}(y_L) = 2.533$  ;  $\theta_{p2} = 68.5^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 163.4^\circ$  ;  $\text{Im}(y_L) = -2.533 + (-2.735) = -5.267$ ;  $\theta_{p1} = 100.7^\circ$  ;  $\theta_{S1} = 129.6^\circ$  ;  
d2)  $\theta_{L2} = 21.7^\circ$  ;  $\text{Im}(y_L) = 2.533 + (-2.735) = -0.202$ ;  $\theta_{p2} = 168.6^\circ$  ;  $\theta_{S1} = 129.6^\circ$  ;  
d3)  $\theta_{L1} = 163.4^\circ$  ;  $\text{Im}(y_L) = -2.533 + (2.735) = 0.202$ ;  $\theta_{p3} = 11.4^\circ$  ;  $\theta_{S2} = 165.8^\circ$  ;  
d4)  $\theta_{L2} = 21.7^\circ$  ;  $\text{Im}(y_L) = 2.533 + (2.735) = 5.267$ ;  $\theta_{p4} = 79.3^\circ$  ;  $\theta_{S2} = 165.8^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 163.4 + 129.6 = 293.0$  ;  $\theta_p = 100.7$  ;  $A \sim 29524.2$   
e2)  $\theta_s = 21.7 + 129.6 = 151.3$  ;  $\theta_p = 168.6$  ;  $A \sim 25514.5$   
e3)  $\theta_s = 163.4 + 165.8 = 329.2$  ;  $\theta_p = 11.4$  ;  $A \sim 3757.0$   
e4)  $\theta_s = 21.7 + 165.8 = 187.5$  ;  $\theta_p = 79.3$  ;  $A \sim 14861.2$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 13

1.  $y = Y/Y_0 = Z_0/Z = 60\Omega / (60.6 - j \cdot 60.9)\Omega = 0.493 + j \cdot 0.495$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0283 + j \cdot 0.0144)] / (0.02 + 0.0283 + j \cdot 0.0144)$   
 $\Gamma = (-0.239) + j \cdot (-0.227) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.330 \angle -136.6^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 23.50\text{dB}$

$P_{in} = 3.15\text{mW} = 4.983\text{dBm}$ ;  $P_{is} = P_{in} - I = 4.983\text{dBm} - 23.50\text{dB} = -18.52\text{dBm} = 14.071\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.613$ ,  $Z_{CE} = 102.09\Omega$ ,  $Z_{CO} = 24.49\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 29)\Omega} = 38.08\Omega$

b)  $Z_L = 29\Omega$  series with  $0.60\text{nH}$  inductor at  $8.4\text{GHz} = 29.00\Omega + j \cdot (31.67)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{in} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 22.81\Omega + j \cdot (-24.90)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 17.50\text{dB}$ ):  $G = G_1 + G_4 = 6.1 + 11.6 = 17.7\text{dB}$ ;  $G = G_2 + G_3 = 8.8 + 9.8 = 18.6\text{dB}$ ;  $G = G_2 + G_4 = 8.8 + 11.6 = 20.4\text{dB}$ ;  $G = G_3 + G_4 = 9.8 + 11.6 = 21.4\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.52\text{dB} = 1.127$ ,  $F_2 = 0.85\text{dB} = 1.216$ ,  $F_3 = 1.06\text{dB} = 1.276$ ,  $F_4 = 1.10\text{dB} = 1.288$ ,  $G_1 = 6.1\text{dB} = 4.074$ ,  $G_2 = 8.8\text{dB} = 7.586$ ;  $F(1,4) = 1.127 + (1.288 - 1)/4.074 = 1.198 = 0.78\text{dB}$ ;  $F(2,3) = 1.216 + (1.276 - 1)/7.586 = 1.254 = 0.98\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.640 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.187 > 1$  ;  $|\Delta| = |(0.259) + j \cdot (0.107)| = 0.280 < 1$

b\_1)  $G_{Tmax} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 10.43 = 10.18\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.028$  ;  $C_1 = (-0.380) + j \cdot (0.332)$  ;  $\Gamma_S = (-0.620) + j \cdot (-0.542) = 0.823 \angle -138.8^\circ$

$B_2 = 0.814$  ;  $C_2 = (-0.371) + j \cdot (-0.134)$  ;  $\Gamma_L = (-0.734) + j \cdot (0.265) = 0.781 \angle 160.1^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 142.1^\circ$  ;  $\text{Im}(y_S) = -2.900$  ;  $\theta_{p1} = 109.0^\circ$  or  $\theta_{S2} = 176.7^\circ$  ;  $\text{Im}(y_S) = 2.900$  ;  $\theta_{p2} = 71.0^\circ$

output:  $\theta_{L1} = 170.6^\circ$  ;  $\text{Im}(y_L) = -2.499$  ;  $\theta_{p1} = 111.8^\circ$  or  $\theta_{L2} = 29.3^\circ$  ;  $\text{Im}(y_L) = 2.499$  ;  $\theta_{p2} = 68.2^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 170.6^\circ$  ;  $\text{Im}(y_L) = -2.499 + (-2.900) = -5.399$ ;  $\theta_{p1} = 100.5^\circ$  ;  $\theta_{S1} = 142.1^\circ$  ;

d2)  $\theta_{L2} = 29.3^\circ$  ;  $\text{Im}(y_L) = 2.499 + (-2.900) = -0.402$ ;  $\theta_{p2} = 158.1^\circ$  ;  $\theta_{S1} = 142.1^\circ$  ;

d3)  $\theta_{L1} = 170.6^\circ$  ;  $\text{Im}(y_L) = -2.499 + (2.900) = 0.402$ ;  $\theta_{p3} = 21.9^\circ$  ;  $\theta_{S2} = 176.7^\circ$  ;

d4)  $\theta_{L2} = 29.3^\circ$  ;  $\text{Im}(y_L) = 2.499 + (2.900) = 5.399$ ;  $\theta_{p4} = 79.5^\circ$  ;  $\theta_{S2} = 176.7^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 170.6 + 142.1 = 312.7$  ;  $\theta_p = 100.5$  ;  $A \sim 31426.9$

e2)  $\theta_s = 29.3 + 142.1 = 171.4$  ;  $\theta_p = 158.1$  ;  $A \sim 27101.5$

e3)  $\theta_s = 170.6 + 176.7 = 347.3$  ;  $\theta_p = 21.9$  ;  $A \sim 7599.9$

e4)  $\theta_s = 29.3 + 176.7 = 206.0$  ;  $\theta_p = 79.5$  ;  $A \sim 16377.5$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 14

1.  $y = Y/Y_0 = Z_0/Z = 100\Omega / (53.9 - j \cdot 68.9)\Omega = 0.704 + j \cdot 0.900$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0337 - j \cdot 0.0358)] / (0.02 + 0.0337 - j \cdot 0.0358)$   
 $\Gamma = (-0.484) + j \cdot (0.344) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.594 \angle 144.6^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless quadrature coupler, matched at the input; Isolation I = 22.80dB

$P_{in} = 2.90mW = 4.624dBm$ ;  $P_{is} = P_{in} - I = 4.624dBm - 22.80dB = -18.18dBm = 15.219\mu W$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.510$ ,  $y_2 = 1.162$ ,  $y_1 = 0.593$ ,  $Z_1 = Z_0/y_1 = 84.3 \Omega$ ,  $Z_2 = Z_0/y_2 = 43.0\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 29)\Omega} = 38.08\Omega$

b)  $Z_L = 29\Omega$  series with 0.46pF capacitor at 7.1GHz =  $29.00\Omega + j \cdot (-48.73)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{in} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 13.08\Omega + j \cdot (21.97)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 17.00dB$ ):  $G = G_1 + G_4 = 6.5 + 10.6 = 17.1dB$ ;  $G = G_2 + G_3 = 8.8 + 9.7 = 18.5dB$ ;  $G = G_2 + G_4 = 8.8 + 10.6 = 19.4dB$ ;  $G = G_3 + G_4 = 9.7 + 10.6 = 20.3dB$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.65dB = 1.161$ ,  $F_2 = 0.73dB = 1.183$ ,  $F_3 = 1.05dB = 1.274$ ,  $F_4 = 1.14dB = 1.300$ ,  $G_1 = 6.5dB = 4.467$ ,  $G_2 = 8.8dB = 7.586$ ;  $F(1,4) = 1.161 + (1.300 - 1)/4.467 = 1.229 = 0.89dB$ ;  $F(2,3) = 1.183 + (1.274 - 1)/7.586 = 1.223 = 0.87dB$ ;

$F(1,4) > F(2,3) \rightarrow$  For minimal noise factor we must use devices 2,3

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.637 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.201 > 1$  ;  $|\Delta| = |(0.265) + j \cdot (0.060)| = 0.272 < 1$

b\_1)  $G_{Tmax} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 10.08 = 10.03dB$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.029$  ;  $C_1 = (-0.351) + j \cdot (0.363)$  ;  $\Gamma_S = (-0.570) + j \cdot (-0.589) = 0.819 \angle -134.1^\circ$

$B_2 = 0.823$  ;  $C_2 = (-0.381) + j \cdot (-0.116)$  ;  $\Gamma_L = (-0.744) + j \cdot (0.227) = 0.778 \angle 163.0^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 139.5^\circ$  ;  $\text{Im}(y_S) = -2.858$  ;  $\theta_{p1} = 109.3^\circ$  or  $\theta_{S2} = 174.5^\circ$  ;  $\text{Im}(y_S) = 2.858$  ;  $\theta_{p2} = 70.7^\circ$

output:  $\theta_{L1} = 169.0^\circ$  ;  $\text{Im}(y_L) = -2.475$  ;  $\theta_{p1} = 112.0^\circ$  or  $\theta_{L2} = 27.9^\circ$  ;  $\text{Im}(y_L) = 2.475$  ;  $\theta_{p2} = 68.0^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 169.0^\circ$  ;  $\text{Im}(y_L) = -2.475 + (-2.858) = -5.333$ ;  $\theta_{p1} = 100.6^\circ$  ;  $\theta_{S1} = 139.5^\circ$  ;

d2)  $\theta_{L2} = 27.9^\circ$  ;  $\text{Im}(y_L) = 2.475 + (-2.858) = -0.383$ ;  $\theta_{p2} = 159.1^\circ$  ;  $\theta_{S1} = 139.5^\circ$  ;

d3)  $\theta_{L1} = 169.0^\circ$  ;  $\text{Im}(y_L) = -2.475 + (2.858) = 0.383$ ;  $\theta_{p3} = 20.9^\circ$  ;  $\theta_{S2} = 174.5^\circ$  ;

d4)  $\theta_{L2} = 27.9^\circ$  ;  $\text{Im}(y_L) = 2.475 + (2.858) = 5.333$ ;  $\theta_{p4} = 79.4^\circ$  ;  $\theta_{S2} = 174.5^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{serie} \times \theta_{paralel} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{serie} \times \theta_{paralel}$ .

e1)  $\theta_s = 169.0 + 139.5 = 308.5$  ;  $\theta_p = 100.6$  ;  $A \sim 31045.6$

e2)  $\theta_s = 27.9 + 139.5 = 167.5$  ;  $\theta_p = 159.1$  ;  $A \sim 26640.4$

e3)  $\theta_s = 169.0 + 174.5 = 343.5$  ;  $\theta_p = 20.9$  ;  $A \sim 7191.6$

e4)  $\theta_s = 27.9 + 174.5 = 202.5$  ;  $\theta_p = 79.4$  ;  $A \sim 16071.8$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 15

1.  $y = Y/Y_0 = Z_0/Z = 100\Omega / (54.6 + j \cdot 52.3)\Omega = 0.955 - j \cdot 0.915$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0240 + j \cdot 0.0187)] / (0.02 + 0.0240 + j \cdot 0.0187)$   
 $\Gamma = (-0.230) + j \cdot (-0.327) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.400 \angle -125.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 21.10\text{dB}$

$P_{in} = 2.05\text{mW} = 3.118\text{dBm}$ ;  $P_{is} = P_{in} - I = 3.118\text{dBm} - 21.10\text{dB} = -17.98\text{dBm} = 15.913\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.465$ ,  $Z_{CE} = 82.74\Omega$ ,  $Z_{CO} = 30.21\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 59)\Omega} = 54.31\Omega$

b)  $Z_L = 59\Omega$  parallel with  $0.91\text{nH}$  inductor at  $9.0\text{GHz} = 25.49\Omega + j \cdot (29.23)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{in} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (-57.33)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.25\text{dB}$ ):  $G = G_1 + G_4 = 6.7 + 10.0 = 16.7\text{dB}$ ;  $G = G_2 + G_3 = 8.9 + 9.5 = 18.4\text{dB}$ ;  $G = G_2 + G_4 = 8.9 + 10.0 = 18.9\text{dB}$ ;  $G = G_3 + G_4 = 9.5 + 10.0 = 19.5\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.66\text{dB} = 1.164$ ,  $F_2 = 0.75\text{dB} = 1.189$ ,  $F_3 = 0.99\text{dB} = 1.256$ ,  $F_4 = 1.13\text{dB} = 1.297$ ,  $G_1 = 6.7\text{dB} = 4.677$ ,  $G_2 = 8.9\text{dB} = 7.762$ ;  $F(1,4) = 1.164 + (1.297 - 1)/4.677 = 1.228 = 0.89\text{dB}$ ;  $F(2,3) = 1.189 + (1.256 - 1)/7.762 = 1.227 = 0.89\text{dB}$ ;

$F(1,4) > F(2,3) \rightarrow$  For minimal noise factor we must use devices 2,3

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.630 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.231 > 1$  ;  $|\Delta| = |(0.255) + j \cdot (-0.019)| = 0.256 < 1$

b\_1)  $G_{Tmax} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 9.41 = 9.74\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.029$  ;  $C_1 = (-0.294) + j \cdot (0.408)$  ;  $\Gamma_S = (-0.473) + j \cdot (-0.657) = 0.810 \angle -125.8^\circ$

$B_2 = 0.840$  ;  $C_2 = (-0.397) + j \cdot (-0.085)$  ;  $\Gamma_L = (-0.754) + j \cdot (0.162) = 0.771 \angle 167.9^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 134.9^\circ$  ;  $\text{Im}(y_S) = -2.762$  ;  $\theta_{p1} = 109.9^\circ$  or  $\theta_{S2} = 170.8^\circ$  ;  $\text{Im}(y_S) = 2.762$  ;  $\theta_{p2} = 70.1^\circ$

output:  $\theta_{L1} = 166.3^\circ$  ;  $\text{Im}(y_L) = -2.421$  ;  $\theta_{p1} = 112.4^\circ$  or  $\theta_{L2} = 25.8^\circ$  ;  $\text{Im}(y_L) = 2.421$  ;  $\theta_{p2} = 67.6^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 166.3^\circ$  ;  $\text{Im}(y_L) = -2.421 + (-2.762) = -5.183$ ;  $\theta_{p1} = 100.9^\circ$  ;  $\theta_{S1} = 134.9^\circ$  ;

d2)  $\theta_{L2} = 25.8^\circ$  ;  $\text{Im}(y_L) = 2.421 + (-2.762) = -0.341$ ;  $\theta_{p2} = 161.2^\circ$  ;  $\theta_{S1} = 134.9^\circ$  ;

d3)  $\theta_{L1} = 166.3^\circ$  ;  $\text{Im}(y_L) = -2.421 + (2.762) = 0.341$ ;  $\theta_{p3} = 18.8^\circ$  ;  $\theta_{S2} = 170.8^\circ$  ;

d4)  $\theta_{L2} = 25.8^\circ$  ;  $\text{Im}(y_L) = 2.421 + (2.762) = 5.183$ ;  $\theta_{p4} = 79.1^\circ$  ;  $\theta_{S2} = 170.8^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{serie} \times \theta_{paralel} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{serie} \times \theta_{paralel}$ .

e1)  $\theta_s = 166.3 + 134.9 = 301.2$  ;  $\theta_p = 100.9$  ;  $A \sim 30397.8$

e2)  $\theta_s = 25.8 + 134.9 = 160.8$  ;  $\theta_p = 161.2$  ;  $A \sim 25911.5$

e3)  $\theta_s = 166.3 + 170.8 = 337.1$  ;  $\theta_p = 18.8$  ;  $A \sim 6345.9$

e4)  $\theta_s = 25.8 + 170.8 = 196.7$  ;  $\theta_p = 79.1$  ;  $A \sim 15553.0$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 16

1.  $y = Y/Y_0 = Z_0/Z = 70\Omega / (68.4 - j \cdot 60.3)\Omega = 0.576 + j \cdot 0.508$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0210 + j \cdot 0.0388)] / (0.02 + 0.0210 + j \cdot 0.0388)$   
 $\Gamma = (-0.485) + j \cdot (-0.487) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.688 \angle -134.9^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = D + C = 25.45\text{dB}$

$P_{\text{in}} = 2.05\text{mW} = 3.118\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 3.118\text{dBm} - 25.45\text{dB} = -22.33\text{dBm} = 5.845\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.599$ ,  $Z_{\text{CE}} = 99.86\Omega$ ,  $Z_{\text{CO}} = 25.04\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 72)\Omega} = 60.00\Omega$

b)  $Z_L = 72\Omega$  series with  $0.67\text{nH}$  inductor at  $9.0\text{GHz} = 72.00\Omega + j \cdot (37.89)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 39.16\Omega + j \cdot (-20.61)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.80\text{dB}$ ):  $G = G_1 + G_4 = 5.2 + 11.0 = 16.2\text{dB}$ ;  $G = G_2 + G_3 = 7.7 + 8.9 = 16.6\text{dB}$ ;  $G = G_2 + G_4 = 7.7 + 11.0 = 18.7\text{dB}$ ;  $G = G_3 + G_4 = 8.9 + 11.0 = 19.9\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.60\text{dB} = 1.148$ ,  $F_2 = 0.77\text{dB} = 1.194$ ,  $F_3 = 1.01\text{dB} = 1.262$ ,  $F_4 = 1.12\text{dB} = 1.294$ ,  $G_1 = 5.2\text{dB} = 3.311$ ,  $G_2 = 7.7\text{dB} = 5.888$ ;  $F(1,4) = 1.148 + (1.294 - 1)/3.311 = 1.237 = 0.92\text{dB}$ ;  $F(2,3) = 1.194 + (1.262 - 1)/5.888 = 1.244 = 0.95\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.599 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.191 > 1$  ;  $|\Delta| = |(-0.015) + j \cdot (-0.257)| = 0.258 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 15.60 = 11.93\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.022$  ;  $C_1 = (-0.476) + j \cdot (0.142)$  ;  $\Gamma_S = (-0.756) + j \cdot (-0.226) = 0.789 \angle -163.4^\circ$

$B_2 = 0.845$  ;  $C_2 = (-0.251) + j \cdot (-0.318)$  ;  $\Gamma_L = (-0.464) + j \cdot (0.588) = 0.749 \angle 128.3^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 152.7^\circ$  ;  $\text{Im}(y_S) = -2.569$  ;  $\theta_{p1} = 111.3^\circ$  or  $\theta_{S2} = 10.6^\circ$  ;  $\text{Im}(y_S) = 2.569$  ;  $\theta_{p2} = 68.7^\circ$

output:  $\theta_{L1} = 5.1^\circ$  ;  $\text{Im}(y_L) = -2.261$  ;  $\theta_{p1} = 113.9^\circ$  or  $\theta_{L2} = 46.6^\circ$  ;  $\text{Im}(y_L) = 2.261$  ;  $\theta_{p2} = 66.1^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 5.1^\circ$  ;  $\text{Im}(y_L) = -2.261 + (-2.569) = -4.831$ ;  $\theta_{p1} = 101.7^\circ$  ;  $\theta_{S1} = 152.7^\circ$  ;

d2)  $\theta_{L2} = 46.6^\circ$  ;  $\text{Im}(y_L) = 2.261 + (-2.569) = -0.308$ ;  $\theta_{p2} = 162.9^\circ$  ;  $\theta_{S1} = 152.7^\circ$  ;

d3)  $\theta_{L1} = 5.1^\circ$  ;  $\text{Im}(y_L) = -2.261 + (2.569) = 0.308$ ;  $\theta_{p3} = 17.1^\circ$  ;  $\theta_{S2} = 10.6^\circ$  ;

d4)  $\theta_{L2} = 46.6^\circ$  ;  $\text{Im}(y_L) = 2.261 + (2.569) = 4.831$ ;  $\theta_{p4} = 78.3^\circ$  ;  $\theta_{S2} = 10.6^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 5.1 + 152.7 = 157.9$  ;  $\theta_p = 101.7$  ;  $A \sim 16053.6$

e2)  $\theta_s = 46.6 + 152.7 = 199.3$  ;  $\theta_p = 162.9$  ;  $A \sim 32469.7$

e3)  $\theta_s = 5.1 + 10.6 = 15.8$  ;  $\theta_p = 17.1$  ;  $A \sim 269.8$

e4)  $\theta_s = 46.6 + 10.6 = 57.2$  ;  $\theta_p = 78.3$  ;  $A \sim 4482.7$

Smallest substrate area is occupied by solution e3 (d3)



## Subject no. 17

1.  $y = Y/Y_0 = Z_0/Z = 90\Omega / (41.2 + j \cdot 37.4)\Omega = 1.198 - j \cdot 1.087$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0195 + j \cdot 0.0299)] / (0.02 + 0.0195 + j \cdot 0.0299)$   
 $\Gamma = (-0.356) + j \cdot (-0.487) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.604 \angle -126.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless quadrature coupler, matched at the input; Isolation  $I = D + C = 28.95\text{dB}$   
 $P_{\text{in}} = 3.10\text{mW} = 4.914\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 4.914\text{dBm} - 28.95\text{dB} = -24.04\text{dBm} = 3.948\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.465$ ,  $y_2 = 1.130$ ,  $y_1 = 0.525$ ,  $Z_1 = Z_0/y_1 = 95.2\Omega$ ,  $Z_2 = Z_0/y_2 = 44.3\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 50)\Omega} = 50.00\Omega$   
b)  $Z_L = 50\Omega$  series with  $0.78\text{nH}$  inductor at  $9.6\text{GHz} = 50.00\Omega + j \cdot (47.05)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 26.52\Omega + j \cdot (-24.95)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 14.95\text{dB}$ ):  $G = G_1 + G_4 = 5.2 + 11.0 = 16.2\text{dB}$ ;  $G = G_2 + G_3 = 8.7 + 8.0 = 16.7\text{dB}$ ;  $G = G_2 + G_4 = 8.7 + 11.0 = 19.7\text{dB}$ ;  $G = G_3 + G_4 = 8.0 + 11.0 = 19.0\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.53\text{dB} = 1.130$ ,  $F_2 = 0.75\text{dB} = 1.189$ ,  $F_3 = 1.05\text{dB} = 1.274$ ,  $F_4 = 1.22\text{dB} = 1.324$ ,  $G_1 = 5.2\text{dB} = 3.311$ ,  
 $G_2 = 8.7\text{dB} = 7.413$ ;  $F(1,4) = 1.130 + (1.324 - 1)/3.311 = 1.228 = 0.89\text{dB}$ ;  $F(2,3) = 1.189 + (1.274 - 1)/7.413 = 1.232 = 0.91\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.603 < 1$  ;  $|S_{22}| = 0.559 < 1$  ;  $K = 1.195 > 1$  ;  $|\Delta| = |(0.229) + j \cdot (-0.082)| = 0.243 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 8.83 = 9.46\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 0.992$  ;  $C_1 = (-0.198) + j \cdot (0.443)$  ;  $\Gamma_S = (-0.329) + j \cdot (-0.736) = 0.807 \angle -114.1^\circ$   
 $B_2 = 0.890$  ;  $C_2 = (-0.431) + j \cdot (-0.032)$  ;  $\Gamma_L = (-0.784) + j \cdot (0.059) = 0.786 \angle 175.7^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 128.9^\circ$  ;  $\text{Im}(y_S) = -2.731$  ;  $\theta_{p1} = 110.1^\circ$  or  $\theta_{S2} = 165.2^\circ$  ;  $\text{Im}(y_S) = 2.731$  ;  $\theta_{p2} = 69.9^\circ$   
output:  $\theta_{L1} = 163.1^\circ$  ;  $\text{Im}(y_L) = -2.546$  ;  $\theta_{p1} = 111.4^\circ$  or  $\theta_{L2} = 21.2^\circ$  ;  $\text{Im}(y_L) = 2.546$  ;  $\theta_{p2} = 68.6^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 163.1^\circ$  ;  $\text{Im}(y_L) = -2.546 + (-2.731) = -5.277$ ;  $\theta_{p1} = 100.7^\circ$  ;  $\theta_{S1} = 128.9^\circ$  ;  
d2)  $\theta_{L2} = 21.2^\circ$  ;  $\text{Im}(y_L) = 2.546 + (-2.731) = -0.185$ ;  $\theta_{p2} = 169.5^\circ$  ;  $\theta_{S1} = 128.9^\circ$  ;  
d3)  $\theta_{L1} = 163.1^\circ$  ;  $\text{Im}(y_L) = -2.546 + (2.731) = 0.185$ ;  $\theta_{p3} = 10.5^\circ$  ;  $\theta_{S2} = 165.2^\circ$  ;  
d4)  $\theta_{L2} = 21.2^\circ$  ;  $\text{Im}(y_L) = 2.546 + (2.731) = 5.277$ ;  $\theta_{p4} = 79.3^\circ$  ;  $\theta_{S2} = 165.2^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 163.1 + 128.9 = 292.0$  ;  $\theta_p = 100.7$  ;  $A \sim 29415.1$   
e2)  $\theta_s = 21.2 + 128.9 = 150.2$  ;  $\theta_p = 169.5$  ;  $A \sim 25458.6$   
e3)  $\theta_s = 163.1 + 165.2 = 328.2$  ;  $\theta_p = 10.5$  ;  $A \sim 3435.4$   
e4)  $\theta_s = 21.2 + 165.2 = 186.4$  ;  $\theta_p = 79.3$  ;  $A \sim 14774.7$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 18

1.  $y = Y/Y_0 = Z_0/Z = 30\Omega / (69.2 - j \cdot 53.4)\Omega = 0.272 + j \cdot 0.210$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0133 - j \cdot 0.0316)] / (0.02 + 0.0133 - j \cdot 0.0316)$   
 $\Gamma = (-0.368) + j \cdot (0.600) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.704 \angle 121.5^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless ring coupler, matched at the input; Isolation  $I = D + C = 31.10\text{dB}$   
 $P_{\text{in}} = 3.45\text{mW} = 5.378\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.378\text{dBm} - 31.10\text{dB} = -25.72\text{dBm} = 2.678\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.473$ ,  $y_1 = 0.473$ ,  $y_2 = 0.881$ ,  $Z_1 = Z_0/y_1 = 105.7\Omega$ ,  $Z_2 = Z_0/y_2 = 56.8\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 71)\Omega} = 59.58\Omega$   
b)  $Z_L = 71\Omega$  parallel with  $0.42\text{pF}$  capacitor at  $9.9\text{GHz} = 15.99\Omega + j \cdot (-29.66)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (92.75)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 14.75\text{dB}$ ):  $G = G_1 + G_4 = 5.9 + 11.1 = 17.0\text{dB}$ ;  $G = G_2 + G_3 = 8.6 + 8.8 = 17.4\text{dB}$ ;  $G = G_2 + G_4 = 8.6 + 11.1 = 19.7\text{dB}$ ;  $G = G_3 + G_4 = 8.8 + 11.1 = 19.9\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.55\text{dB} = 1.135$ ,  $F_2 = 0.86\text{dB} = 1.219$ ,  $F_3 = 0.90\text{dB} = 1.230$ ,  $F_4 = 1.13\text{dB} = 1.297$ ,  $G_1 = 5.9\text{dB} = 3.890$ ,  $G_2 = 8.6\text{dB} = 7.244$ ;  $F(1,4) = 1.135 + (1.297 - 1)/3.890 = 1.211 = 0.83\text{dB}$ ;  $F(2,3) = 1.219 + (1.230 - 1)/7.244 = 1.260 = 1.00\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.608 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.157 > 1$  ;  $|\Delta| = |(-0.048) + j \cdot (0.263)| = 0.267 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 16.74 = 12.24\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.028$  ;  $C_1 = (-0.491) + j \cdot (0.108)$  ;  $\Gamma_S = (-0.788) + j \cdot (-0.173) = 0.807 \angle -167.6^\circ$   
 $B_2 = 0.829$  ;  $C_2 = (-0.238) + j \cdot (-0.322)$  ;  $\Gamma_L = (-0.454) + j \cdot (0.615) = 0.765 \angle 126.4^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 155.7^\circ$  ;  $\text{Im}(y_S) = -2.732$  ;  $\theta_{p1} = 110.1^\circ$  or  $\theta_{S2} = 11.9^\circ$  ;  $\text{Im}(y_S) = 2.732$  ;  $\theta_{p2} = 69.9^\circ$   
output:  $\theta_{L1} = 6.7^\circ$  ;  $\text{Im}(y_L) = -2.373$  ;  $\theta_{p1} = 112.8^\circ$  or  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.373$  ;  $\theta_{p2} = 67.2^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 6.7^\circ$  ;  $\text{Im}(y_L) = -2.373 + (-2.732) = -5.105$ ;  $\theta_{p1} = 101.1^\circ$  ;  $\theta_{S1} = 155.7^\circ$  ;  
d2)  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.373 + (-2.732) = -0.358$ ;  $\theta_{p2} = 160.3^\circ$  ;  $\theta_{S1} = 155.7^\circ$  ;  
d3)  $\theta_{L1} = 6.7^\circ$  ;  $\text{Im}(y_L) = -2.373 + (2.732) = 0.358$ ;  $\theta_{p3} = 19.7^\circ$  ;  $\theta_{S2} = 11.9^\circ$  ;  
d4)  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.373 + (2.732) = 5.105$ ;  $\theta_{p4} = 78.9^\circ$  ;  $\theta_{S2} = 11.9^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 6.7 + 155.7 = 162.4$  ;  $\theta_p = 101.1$  ;  $A \sim 16418.7$   
e2)  $\theta_s = 46.8 + 155.7 = 202.5$  ;  $\theta_p = 160.3$  ;  $A \sim 32468.6$   
e3)  $\theta_s = 6.7 + 11.9 = 18.6$  ;  $\theta_p = 19.7$  ;  $A \sim 367.1$   
e4)  $\theta_s = 46.8 + 11.9 = 58.8$  ;  $\theta_p = 78.9$  ;  $A \sim 4637.0$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 19

1.  $y = Y/Y_0 = Z_0/Z = 70\Omega / (31.2 - j \cdot 59.0)\Omega = 0.490 + j \cdot 0.927$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0249 + j \cdot 0.0350)] / (0.02 + 0.0249 + j \cdot 0.0350)$   
 $\Gamma = (-0.446) + j \cdot (-0.432) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.621 \angle -135.9^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless ring coupler, matched at the input; Isolation  $I = D + C = 26.35\text{dB}$   
 $P_{\text{in}} = 3.30\text{mW} = 5.185\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.185\text{dBm} - 26.35\text{dB} = -21.16\text{dBm} = 7.647\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.585$ ,  $y_1 = 0.585$ ,  $y_2 = 0.811$ ,  $Z_1 = Z_0/y_1 = 85.4\Omega$ ,  $Z_2 = Z_0/y_2 = 61.7\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 74)\Omega} = 60.83\Omega$   
b)  $Z_L = 74\Omega$  series with  $0.73\text{nH}$  inductor at  $8.7\text{GHz} = 74.00\Omega + j \cdot (39.90)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 38.74\Omega + j \cdot (-20.89)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 16.30\text{dB}$ ):  $G = G_1 + G_4 = 5.0 + 11.9 = 16.9\text{dB}$ ;  $G = G_2 + G_3 = 8.2 + 9.4 = 17.6\text{dB}$ ;  $G = G_2 + G_4 = 8.2 + 11.9 = 20.1\text{dB}$ ;  $G = G_3 + G_4 = 9.4 + 11.9 = 21.3\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.66\text{dB} = 1.164$ ,  $F_2 = 0.78\text{dB} = 1.197$ ,  $F_3 = 1.02\text{dB} = 1.265$ ,  $F_4 = 1.19\text{dB} = 1.315$ ,  $G_1 = 5.0\text{dB} = 3.162$ ,  $G_2 = 8.2\text{dB} = 6.607$ ;  $F(1,4) = 1.164 + (1.315 - 1)/3.162 = 1.264 = 1.02\text{dB}$ ;  $F(2,3) = 1.197 + (1.265 - 1)/6.607 = 1.244 = 0.95\text{dB}$ ;  
 $F(1,4) > F(2,3) \rightarrow$  For minimal noise factor we must use devices 2,3
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.620 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.114 > 1$  ;  $|\Delta| = |(-0.096) + j \cdot (0.264)| = 0.281 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 18.51 = 12.67\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.035$  ;  $C_1 = (-0.506) + j \cdot (0.059)$  ;  $\Gamma_S = (-0.827) + j \cdot (-0.097) = 0.833 \angle -173.3^\circ$   
 $B_2 = 0.807$  ;  $C_2 = (-0.220) + j \cdot (-0.325)$  ;  $\Gamma_L = (-0.443) + j \cdot (0.654) = 0.790 \angle 124.1^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 159.9^\circ$  ;  $\text{Im}(y_S) = -3.011$  ;  $\theta_{p1} = 108.4^\circ$  or  $\theta_{S2} = 13.5^\circ$  ;  $\text{Im}(y_S) = 3.011$  ;  $\theta_{p2} = 71.6^\circ$   
output:  $\theta_{L1} = 9.0^\circ$  ;  $\text{Im}(y_L) = -2.574$  ;  $\theta_{p1} = 111.2^\circ$  or  $\theta_{L2} = 46.9^\circ$  ;  $\text{Im}(y_L) = 2.574$  ;  $\theta_{p2} = 68.8^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 9.0^\circ$  ;  $\text{Im}(y_L) = -2.574 + (-3.011) = -5.585$ ;  $\theta_{p1} = 100.2^\circ$  ;  $\theta_{S1} = 159.9^\circ$  ;  
d2)  $\theta_{L2} = 46.9^\circ$  ;  $\text{Im}(y_L) = 2.574 + (-3.011) = -0.436$ ;  $\theta_{p2} = 156.4^\circ$  ;  $\theta_{S1} = 159.9^\circ$  ;  
d3)  $\theta_{L1} = 9.0^\circ$  ;  $\text{Im}(y_L) = -2.574 + (3.011) = 0.436$ ;  $\theta_{p3} = 23.6^\circ$  ;  $\theta_{S2} = 13.5^\circ$  ;  
d4)  $\theta_{L2} = 46.9^\circ$  ;  $\text{Im}(y_L) = 2.574 + (3.011) = 5.585$ ;  $\theta_{p4} = 79.8^\circ$  ;  $\theta_{S2} = 13.5^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 9.0 + 159.9 = 168.9$  ;  $\theta_p = 100.2$  ;  $A \sim 16915.2$   
e2)  $\theta_s = 46.9 + 159.9 = 206.7$  ;  $\theta_p = 156.4$  ;  $A \sim 32339.5$   
e3)  $\theta_s = 9.0 + 13.5 = 22.5$  ;  $\theta_p = 23.6$  ;  $A \sim 530.2$   
e4)  $\theta_s = 46.9 + 13.5 = 60.3$  ;  $\theta_p = 79.8$  ;  $A \sim 4817.6$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 20

1.  $y = Y/Y_0 = Z_0/Z = 50\Omega / (56.6 + j \cdot 36.8)\Omega = 0.621 - j \cdot 0.404$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0105 - j \cdot 0.0151)] / (0.02 + 0.0105 - j \cdot 0.0151)$   
 $\Gamma = (0.053) + j \cdot (0.521) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.524 \angle 84.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = D + C = 31.20\text{dB}$

$P_{\text{in}} = 3.90\text{mW} = 5.911\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.911\text{dBm} - 31.20\text{dB} = -25.29\text{dBm} = 2.958\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.468$ ,  $Z_{\text{CE}} = 83.03\Omega$ ,  $Z_{\text{CO}} = 30.11\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 65)\Omega} = 57.01\Omega$

b)  $Z_L = 65\Omega$  series with  $0.30\text{pF}$  capacitor at  $9.0\text{GHz} = 65.00\Omega + j \cdot (-58.95)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 27.44\Omega + j \cdot (24.88)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.85\text{dB}$ ):  $G = G_1 + G_4 = 6.1 + 11.4 = 17.5\text{dB}$ ;  $G = G_2 + G_3 = 7.2 + 9.9 =$

$17.1\text{dB}$ ;  $G = G_2 + G_4 = 7.2 + 11.4 = 18.6\text{dB}$ ;  $G = G_3 + G_4 = 9.9 + 11.4 = 21.3\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.57\text{dB} = 1.140$ ,  $F_2 = 0.70\text{dB} = 1.175$ ,  $F_3 = 0.94\text{dB} = 1.242$ ,  $F_4 = 1.19\text{dB} = 1.315$ ,  $G_1 = 6.1\text{dB} = 4.074$ ,

$G_2 = 7.2\text{dB} = 5.248$ ;  $F(1,4) = 1.140 + (1.315 - 1)/4.074 = 1.218 = 0.86\text{dB}$ ;  $F(2,3) = 1.175 +$

$(1.242 - 1)/5.248 = 1.235 = 0.92\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.639 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.193 > 1$  ;  $|\Delta| = |(0.264) + j \cdot (0.084)| = 0.277 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 10.26 = 10.11\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.029$  ;  $C_1 = (-0.366) + j \cdot (0.348)$  ;  $\Gamma_S = (-0.595) + j \cdot (-0.566) = 0.822 \angle -136.4^\circ$

$B_2 = 0.818$  ;  $C_2 = (-0.376) + j \cdot (-0.125)$  ;  $\Gamma_L = (-0.740) + j \cdot (0.245) = 0.779 \angle 161.7^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 140.8^\circ$  ;  $\text{Im}(y_S) = -2.883$  ;  $\theta_{p1} = 109.1^\circ$  or  $\theta_{S2} = 175.6^\circ$  ;  $\text{Im}(y_S) = 2.883$  ;  $\theta_{p2} = 70.9^\circ$

output:  $\theta_{L1} = 169.8^\circ$  ;  $\text{Im}(y_L) = -2.488$  ;  $\theta_{p1} = 111.9^\circ$  or  $\theta_{L2} = 28.6^\circ$  ;  $\text{Im}(y_L) = 2.488$  ;  $\theta_{p2} = 68.1^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 169.8^\circ$  ;  $\text{Im}(y_L) = -2.488 + (-2.883) = -5.372$ ;  $\theta_{p1} = 100.5^\circ$  ;  $\theta_{S1} = 140.8^\circ$  ;

d2)  $\theta_{L2} = 28.6^\circ$  ;  $\text{Im}(y_L) = 2.488 + (-2.883) = -0.395$ ;  $\theta_{p2} = 158.5^\circ$  ;  $\theta_{S1} = 140.8^\circ$  ;

d3)  $\theta_{L1} = 169.8^\circ$  ;  $\text{Im}(y_L) = -2.488 + (2.883) = 0.395$ ;  $\theta_{p3} = 21.5^\circ$  ;  $\theta_{S2} = 175.6^\circ$  ;

d4)  $\theta_{L2} = 28.6^\circ$  ;  $\text{Im}(y_L) = 2.488 + (2.883) = 5.372$ ;  $\theta_{p4} = 79.5^\circ$  ;  $\theta_{S2} = 175.6^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 169.8 + 140.8 = 310.6$  ;  $\theta_p = 100.5$  ;  $A \sim 31230.5$

e2)  $\theta_s = 28.6 + 140.8 = 169.4$  ;  $\theta_p = 158.5$  ;  $A \sim 26842.5$

e3)  $\theta_s = 169.8 + 175.6 = 345.4$  ;  $\theta_p = 21.5$  ;  $A \sim 7440.4$

e4)  $\theta_s = 28.6 + 175.6 = 204.1$  ;  $\theta_p = 79.5$  ;  $A \sim 16220.5$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 21

1.  $y = Y/Y_0 = Z_0/Z = 80\Omega / (43.6 + j \cdot 37.4)\Omega = 1.057 - j \cdot 0.907$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0336 + j \cdot 0.0168)] / (0.02 + 0.0336 + j \cdot 0.0168)$   
 $\Gamma = (-0.320) + j \cdot (-0.213) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.385 \angle -146.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless ring coupler, matched at the input; Isolation  $I = D + C = 24.95\text{dB}$

$P_{\text{in}} = 1.85\text{mW} = 2.672\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 2.672\text{dBm} - 24.95\text{dB} = -22.28\text{dBm} = 5.918\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.613$ ,  $y_1 = 0.613$ ,  $y_2 = 0.790$ ,  $Z_1 = Z_0/y_1 = 81.6\Omega$ ,  $Z_2 = Z_0/y_2 = 63.3\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 54)\Omega} = 51.96\Omega$

b)  $Z_L = 54\Omega$  series with  $0.37\text{pF}$  capacitor at  $6.6\text{GHz} = 54.00\Omega + j \cdot (-65.17)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 20.35\Omega + j \cdot (24.56)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.05\text{dB}$ ):  $G = G_1 + G_4 = 6.1 + 11.7 = 17.8\text{dB}$ ;  $G = G_2 + G_3 = 8.5 + 9.4 = 17.9\text{dB}$ ;  $G = G_2 + G_4 = 8.5 + 11.7 = 20.2\text{dB}$ ;  $G = G_3 + G_4 = 9.4 + 11.7 = 21.1\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.67\text{dB} = 1.167$ ,  $F_2 = 0.76\text{dB} = 1.191$ ,  $F_3 = 0.97\text{dB} = 1.250$ ,  $F_4 = 1.20\text{dB} = 1.318$ ,  $G_1 = 6.1\text{dB} = 4.074$ ,  $G_2 = 8.5\text{dB} = 7.079$ ;  $F(1,4) = 1.167 + (1.318 - 1)/4.074 = 1.245 = 0.95\text{dB}$ ;  $F(2,3) = 1.191 + (1.250 - 1)/7.079 = 1.236 = 0.92\text{dB}$ ;

$F(1,4) > F(2,3) \rightarrow$  For minimal noise factor we must use devices 2,3

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.644 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.101 > 1$  ;  $|\Delta| = |(-0.126) + j \cdot (-0.250)| = 0.280 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 22.09 = 13.44\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.066$  ;  $C_1 = (-0.525) + j \cdot (-0.045)$  ;  $\Gamma_S = (-0.853) + j \cdot (0.073) = 0.856 \angle 175.1^\circ$

$B_2 = 0.777$  ;  $C_2 = (-0.211) + j \cdot (-0.316)$  ;  $\Gamma_L = (-0.447) + j \cdot (0.671) = 0.807 \angle 123.7^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 166.9^\circ$  ;  $\text{Im}(y_S) = -3.311$  ;  $\theta_{p1} = 106.8^\circ$  or  $\theta_{S2} = 18.0^\circ$  ;  $\text{Im}(y_S) = 3.311$  ;  $\theta_{p2} = 73.2^\circ$

output:  $\theta_{L1} = 10.0^\circ$  ;  $\text{Im}(y_L) = -2.730$  ;  $\theta_{p1} = 110.1^\circ$  or  $\theta_{L2} = 46.3^\circ$  ;  $\text{Im}(y_L) = 2.730$  ;  $\theta_{p2} = 69.9^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 10.0^\circ$  ;  $\text{Im}(y_L) = -2.730 + (-3.311) = -6.042$ ;  $\theta_{p1} = 99.4^\circ$  ;  $\theta_{S1} = 166.9^\circ$  ;

d2)  $\theta_{L2} = 46.3^\circ$  ;  $\text{Im}(y_L) = 2.730 + (-3.311) = -0.581$ ;  $\theta_{p2} = 149.8^\circ$  ;  $\theta_{S1} = 166.9^\circ$  ;

d3)  $\theta_{L1} = 10.0^\circ$  ;  $\text{Im}(y_L) = -2.730 + (3.311) = 0.581$ ;  $\theta_{p3} = 30.2^\circ$  ;  $\theta_{S2} = 18.0^\circ$  ;

d4)  $\theta_{L2} = 46.3^\circ$  ;  $\text{Im}(y_L) = 2.730 + (3.311) = 6.042$ ;  $\theta_{p4} = 80.6^\circ$  ;  $\theta_{S2} = 18.0^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 10.0 + 166.9 = 176.9$  ;  $\theta_p = 99.4$  ;  $A \sim 17586.7$

e2)  $\theta_s = 46.3 + 166.9 = 213.2$  ;  $\theta_p = 149.8$  ;  $A \sim 31937.7$

e3)  $\theta_s = 10.0 + 18.0 = 28.1$  ;  $\theta_p = 30.2$  ;  $A \sim 846.6$

e4)  $\theta_s = 46.3 + 18.0 = 64.3$  ;  $\theta_p = 80.6$  ;  $A \sim 5181.6$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 22

1.  $y = Y/Y_0 = Z_0/Z = 85\Omega / (44.4 - j \cdot 56.6)\Omega = 0.729 + j \cdot 0.930$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0184 - j \cdot 0.0361)] / (0.02 + 0.0184 - j \cdot 0.0361)$   
 $\Gamma = (-0.447) + j \cdot (0.520) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.686 \angle 130.7^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless quadrature coupler, matched at the input; Isolation I = 21.50dB  
 $P_{in} = 4.05\text{mW} = 6.075\text{dBm}$ ;  $P_{is} = P_{in} - I = 6.075\text{dBm} - 21.50\text{dB} = -15.43\text{dBm} = 28.672\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.572$ ,  $y_2 = 1.219$ ,  $y_1 = 0.698$ ,  $Z_1 = Z_0/y_1 = 71.7 \Omega$ ,  $Z_2 = Z_0/y_2 = 41.0\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 67)\Omega} = 57.88\Omega$   
b)  $Z_L = 67\Omega$  series with 1.05nH inductor at 7.5GHz =  $67.00\Omega + j \cdot (49.48)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{in} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 32.35\Omega + j \cdot (-23.89)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 14.55\text{dB}$ ):  $G = G_1 + G_4 = 5.1 + 11.6 = 16.7\text{dB}$ ;  $G = G_2 + G_3 = 8.7 + 9.4 = 18.1\text{dB}$ ;  $G = G_2 + G_4 = 8.7 + 11.6 = 20.3\text{dB}$ ;  $G = G_3 + G_4 = 9.4 + 11.6 = 21.0\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.51\text{dB} = 1.125$ ,  $F_2 = 0.80\text{dB} = 1.202$ ,  $F_3 = 0.94\text{dB} = 1.242$ ,  $F_4 = 1.21\text{dB} = 1.321$ ,  $G_1 = 5.1\text{dB} = 3.236$ ,  $G_2 = 8.7\text{dB} = 7.413$ ;  $F(1,4) = 1.125 + (1.321 - 1)/3.236 = 1.224 = 0.88\text{dB}$ ;  $F(2,3) = 1.202 + (1.242 - 1)/7.413 = 1.246 = 0.95\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.635 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.105 > 1$  ;  $|\Delta| = |(-0.115) + j \cdot (0.256)| = 0.280 < 1$   
b\_1)  $G_{Tmax} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 20.69 = 13.16\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.054$  ;  $C_1 = (-0.520) + j \cdot (-0.005)$  ;  $\Gamma_S = (-0.848) + j \cdot (0.008) = 0.848 \angle 179.4^\circ$   
 $B_2 = 0.789$  ;  $C_2 = (-0.214) + j \cdot (-0.320)$  ;  $\Gamma_L = (-0.446) + j \cdot (0.665) = 0.801 \angle 123.9^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 164.3^\circ$  ;  $\text{Im}(y_S) = -3.202$  ;  $\theta_{p1} = 107.3^\circ$  or  $\theta_{S2} = 16.3^\circ$  ;  $\text{Im}(y_S) = 3.202$  ;  $\theta_{p2} = 72.7^\circ$   
output:  $\theta_{L1} = 9.7^\circ$  ;  $\text{Im}(y_L) = -2.677$  ;  $\theta_{p1} = 110.5^\circ$  or  $\theta_{L2} = 46.5^\circ$  ;  $\text{Im}(y_L) = 2.677$  ;  $\theta_{p2} = 69.5^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 9.7^\circ$  ;  $\text{Im}(y_L) = -2.677 + (-3.202) = -5.878$ ;  $\theta_{p1} = 99.7^\circ$  ;  $\theta_{S1} = 164.3^\circ$  ;  
d2)  $\theta_{L2} = 46.5^\circ$  ;  $\text{Im}(y_L) = 2.677 + (-3.202) = -0.525$ ;  $\theta_{p2} = 152.3^\circ$  ;  $\theta_{S1} = 164.3^\circ$  ;  
d3)  $\theta_{L1} = 9.7^\circ$  ;  $\text{Im}(y_L) = -2.677 + (3.202) = 0.525$ ;  $\theta_{p3} = 27.7^\circ$  ;  $\theta_{S2} = 16.3^\circ$  ;  
d4)  $\theta_{L2} = 46.5^\circ$  ;  $\text{Im}(y_L) = 2.677 + (3.202) = 5.878$ ;  $\theta_{p4} = 80.3^\circ$  ;  $\theta_{S2} = 16.3^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{serie} \times \theta_{paralel} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{serie} \times \theta_{paralel}$ .  
e1)  $\theta_s = 9.7 + 164.3 = 174.0$  ;  $\theta_p = 99.7$  ;  $A \sim 17337.3$   
e2)  $\theta_s = 46.5 + 164.3 = 210.7$  ;  $\theta_p = 152.3$  ;  $A \sim 32095.0$   
e3)  $\theta_s = 9.7 + 16.3 = 26.0$  ;  $\theta_p = 27.7$  ;  $A \sim 719.4$   
e4)  $\theta_s = 46.5 + 16.3 = 62.7$  ;  $\theta_p = 80.3$  ;  $A \sim 5040.3$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 23

1.  $y = Y/Y_0 = Z_0/Z = 40\Omega / (30.7 + j \cdot 53.0)\Omega = 0.327 - j \cdot 0.565$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0315 - j \cdot 0.0161)] / (0.02 + 0.0315 - j \cdot 0.0161)$   
 $\Gamma = (-0.292) + j \cdot (0.221) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.367 \angle 142.9^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 24.50\text{dB}$

$P_{\text{in}} = 2.80\text{mW} = 4.472\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 4.472\text{dBm} - 24.50\text{dB} = -20.03\text{dBm} = 9.935\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.631$ ,  $Z_{\text{CE}} = 105.11\Omega$ ,  $Z_{\text{CO}} = 23.78\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 46)\Omega} = 47.96\Omega$

b)  $Z_L = 46\Omega$  series with  $0.30\text{pF}$  capacitor at  $9.3\text{GHz} = 46.00\Omega + j \cdot (-57.04)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 19.70\Omega + j \cdot (24.43)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.10\text{dB}$ ):  $G = G_1 + G_4 = 5.6 + 10.3 = 15.9\text{dB}$ ;  $G = G_2 + G_3 = 8.5 + 9.1 = 17.6\text{dB}$ ;  $G = G_2 + G_4 = 8.5 + 10.3 = 18.8\text{dB}$ ;  $G = G_3 + G_4 = 9.1 + 10.3 = 19.4\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.64\text{dB} = 1.159$ ,  $F_2 = 0.82\text{dB} = 1.208$ ,  $F_3 = 0.98\text{dB} = 1.253$ ,  $F_4 = 1.19\text{dB} = 1.315$ ,  $G_1 = 5.6\text{dB} = 3.631$ ,  
 $G_2 = 8.5\text{dB} = 7.079$ ;  $F(1,4) = 1.159 + (1.315 - 1)/3.631 = 1.246 = 0.95\text{dB}$ ;  $F(2,3) = 1.208 + (1.253 - 1)/7.079 = 1.252 = 0.98\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.633 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.218 > 1$  ;  $|\Delta| = |(0.262) + j \cdot (0.015)| = 0.263 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 9.69 = 9.87\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.029$  ;  $C_1 = (-0.319) + j \cdot (0.390)$  ;  $\Gamma_S = (-0.516) + j \cdot (-0.630) = 0.814 \angle -129.3^\circ$

$B_2 = 0.833$  ;  $C_2 = (-0.391) + j \cdot (-0.099)$  ;  $\Gamma_L = (-0.750) + j \cdot (0.190) = 0.774 \angle 165.8^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 136.9^\circ$  ;  $\text{Im}(y_S) = -2.803$  ;  $\theta_{p1} = 109.6^\circ$  or  $\theta_{S2} = 172.4^\circ$  ;  $\text{Im}(y_S) = 2.803$  ;  $\theta_{p2} = 70.4^\circ$

output:  $\theta_{L1} = 167.5^\circ$  ;  $\text{Im}(y_L) = -2.445$  ;  $\theta_{p1} = 112.2^\circ$  or  $\theta_{L2} = 26.7^\circ$  ;  $\text{Im}(y_L) = 2.445$  ;  $\theta_{p2} = 67.8^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 167.5^\circ$  ;  $\text{Im}(y_L) = -2.445 + (-2.803) = -5.248$ ;  $\theta_{p1} = 100.8^\circ$  ;  $\theta_{S1} = 136.9^\circ$  ;

d2)  $\theta_{L2} = 26.7^\circ$  ;  $\text{Im}(y_L) = 2.445 + (-2.803) = -0.358$ ;  $\theta_{p2} = 160.3^\circ$  ;  $\theta_{S1} = 136.9^\circ$  ;

d3)  $\theta_{L1} = 167.5^\circ$  ;  $\text{Im}(y_L) = -2.445 + (2.803) = 0.358$ ;  $\theta_{p3} = 19.7^\circ$  ;  $\theta_{S2} = 172.4^\circ$  ;

d4)  $\theta_{L2} = 26.7^\circ$  ;  $\text{Im}(y_L) = 2.445 + (2.803) = 5.248$ ;  $\theta_{p4} = 79.2^\circ$  ;  $\theta_{S2} = 172.4^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 167.5 + 136.9 = 304.4$  ;  $\theta_p = 100.8$  ;  $A \sim 30676.0$

e2)  $\theta_s = 26.7 + 136.9 = 163.6$  ;  $\theta_p = 160.3$  ;  $A \sim 26229.3$

e3)  $\theta_s = 167.5 + 172.4 = 339.9$  ;  $\theta_p = 19.7$  ;  $A \sim 6701.9$

e4)  $\theta_s = 26.7 + 172.4 = 199.2$  ;  $\theta_p = 79.2$  ;  $A \sim 15775.2$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 24

1.  $y = Y/Y_0 = Z_0/Z = 50\Omega / (49.9 + j \cdot 60.2)\Omega = 0.408 - j \cdot 0.492$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0327 - j \cdot 0.0257)] / (0.02 + 0.0327 - j \cdot 0.0257)$   
 $\Gamma = (-0.387) + j \cdot (0.299) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.489 \angle 142.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 21.00\text{dB}$

$P_{\text{in}} = 2.70\text{mW} = 4.314\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 4.314\text{dBm} - 21.00\text{dB} = -16.69\text{dBm} = 21.447\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.631$ ,  $Z_{\text{CE}} = 105.11\Omega$ ,  $Z_{\text{CO}} = 23.78\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 70)\Omega} = 59.16\Omega$

b)  $Z_L = 70\Omega$  parallel with  $1.39\text{nH}$  inductor at  $7.7\text{GHz} = 33.60\Omega + j \cdot (34.97)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (-52.05)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.60\text{dB}$ ):  $G = G_1 + G_4 = 6.0 + 11.3 = 17.3\text{dB}$ ;  $G = G_2 + G_3 = 8.6 + 8.5 = 17.1\text{dB}$ ;  $G = G_2 + G_4 = 8.6 + 11.3 = 19.9\text{dB}$ ;  $G = G_3 + G_4 = 8.5 + 11.3 = 19.8\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.54\text{dB} = 1.132$ ,  $F_2 = 0.72\text{dB} = 1.180$ ,  $F_3 = 0.94\text{dB} = 1.242$ ,  $F_4 = 1.15\text{dB} = 1.303$ ,  $G_1 = 6.0\text{dB} = 3.981$ ,  
 $G_2 = 8.6\text{dB} = 7.244$ ;  $F(1,4) = 1.132 + (1.303 - 1)/3.981 = 1.209 = 0.82\text{dB}$ ;  $F(2,3) = 1.180 + (1.242 - 1)/7.244 = 1.222 = 0.87\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.600 < 1$  ;  $|S_{22}| = 0.560 < 1$  ;  $K = 1.192 > 1$  ;  $|\Delta| = |(0.225) + j \cdot (-0.088)| = 0.242 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 8.77 = 9.43\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 0.988$  ;  $C_1 = (-0.187) + j \cdot (0.445)$  ;  $\Gamma_S = (-0.313) + j \cdot (-0.743) = 0.806 \angle -112.8^\circ$

$B_2 = 0.895$  ;  $C_2 = (-0.434) + j \cdot (-0.026)$  ;  $\Gamma_L = (-0.787) + j \cdot (0.047) = 0.788 \angle 176.6^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 128.3^\circ$  ;  $\text{Im}(y_S) = -2.727$  ;  $\theta_{p1} = 110.1^\circ$  or  $\theta_{S2} = 164.5^\circ$  ;  $\text{Im}(y_S) = 2.727$  ;  $\theta_{p2} = 69.9^\circ$

output:  $\theta_{L1} = 162.7^\circ$  ;  $\text{Im}(y_L) = -2.560$  ;  $\theta_{p1} = 111.3^\circ$  or  $\theta_{L2} = 20.7^\circ$  ;  $\text{Im}(y_L) = 2.560$  ;  $\theta_{p2} = 68.7^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 162.7^\circ$  ;  $\text{Im}(y_L) = -2.560 + (-2.727) = -5.287$ ;  $\theta_{p1} = 100.7^\circ$  ;  $\theta_{S1} = 128.3^\circ$  ;

d2)  $\theta_{L2} = 20.7^\circ$  ;  $\text{Im}(y_L) = 2.560 + (-2.727) = -0.168$ ;  $\theta_{p2} = 170.5^\circ$  ;  $\theta_{S1} = 128.3^\circ$  ;

d3)  $\theta_{L1} = 162.7^\circ$  ;  $\text{Im}(y_L) = -2.560 + (2.727) = 0.168$ ;  $\theta_{p3} = 9.5^\circ$  ;  $\theta_{S2} = 164.5^\circ$  ;

d4)  $\theta_{L2} = 20.7^\circ$  ;  $\text{Im}(y_L) = 2.560 + (2.727) = 5.287$ ;  $\theta_{p4} = 79.3^\circ$  ;  $\theta_{S2} = 164.5^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 162.7 + 128.3 = 291.0$  ;  $\theta_p = 100.7$  ;  $A \sim 29306.1$

e2)  $\theta_s = 20.7 + 128.3 = 149.0$  ;  $\theta_p = 170.5$  ;  $A \sim 25400.9$

e3)  $\theta_s = 162.7 + 164.5 = 327.2$  ;  $\theta_p = 9.5$  ;  $A \sim 3114.9$

e4)  $\theta_s = 20.7 + 164.5 = 185.2$  ;  $\theta_p = 79.3$  ;  $A \sim 14688.2$

Smallest substrate area is occupied by solution e3 (d3)



## Subject no. 25

1.  $y = Y/Y_0 = Z_0/Z = 30\Omega / (35.4 + j \cdot 58.9)\Omega = 0.225 - j \cdot 0.374$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0342 + j \cdot 0.0358)] / (0.02 + 0.0342 + j \cdot 0.0358)$   
 $\Gamma = (-0.486) + j \cdot (-0.339) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.593 \angle -145.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless quadrature coupler, matched at the input; Isolation  $I = 22.90\text{dB}$   
 $P_{\text{in}} = 1.75\text{mW} = 2.430\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 2.430\text{dBm} - 22.90\text{dB} = -20.47\text{dBm} = 8.975\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.490$ ,  $y_2 = 1.147$ ,  $y_1 = 0.562$ ,  $Z_1 = Z_0/y_1 = 89.0\Omega$ ,  $Z_2 = Z_0/y_2 = 43.6\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 28)\Omega} = 37.42\Omega$   
b)  $Z_L = 28\Omega$  series with  $1.44\text{nH}$  inductor at  $7.5\text{GHz} = 28.00\Omega + j \cdot (67.86)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 7.27\Omega + j \cdot (-17.63)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 14.65\text{dB}$ ):  $G = G_1 + G_4 = 5.1 + 11.9 = 17.0\text{dB}$ ;  $G = G_2 + G_3 = 8.2 + 8.6 = 16.8\text{dB}$ ;  $G = G_2 + G_4 = 8.2 + 11.9 = 20.1\text{dB}$ ;  $G = G_3 + G_4 = 8.6 + 11.9 = 20.5\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.64\text{dB} = 1.159$ ,  $F_2 = 0.81\text{dB} = 1.205$ ,  $F_3 = 0.98\text{dB} = 1.253$ ,  $F_4 = 1.25\text{dB} = 1.334$ ,  $G_1 = 5.1\text{dB} = 3.236$ ,  $G_2 = 8.2\text{dB} = 6.607$ ;  $F(1,4) = 1.159 + (1.334 - 1)/3.236 = 1.262 = 1.01\text{dB}$ ;  $F(2,3) = 1.205 + (1.253 - 1)/6.607 = 1.256 = 0.99\text{dB}$ ;  
 $F(1,4) > F(2,3) \rightarrow$  For minimal noise factor we must use devices 2,3
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.623 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.112 > 1$  ;  $|\Delta| = |(-0.100) + j \cdot (-0.262)| = 0.281 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 18.93 = 12.77\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.039$  ;  $C_1 = (-0.509) + j \cdot (0.047)$  ;  $\Gamma_S = (-0.833) + j \cdot (-0.076) = 0.836 \angle -174.8^\circ$   
 $B_2 = 0.804$  ;  $C_2 = (-0.219) + j \cdot (-0.324)$  ;  $\Gamma_L = (-0.443) + j \cdot (0.656) = 0.792 \angle 124.0^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 160.8^\circ$  ;  $\text{Im}(y_S) = -3.049$  ;  $\theta_{p1} = 108.2^\circ$  or  $\theta_{S2} = 14.0^\circ$  ;  $\text{Im}(y_S) = 3.049$  ;  $\theta_{p2} = 71.8^\circ$   
output:  $\theta_{L1} = 9.2^\circ$  ;  $\text{Im}(y_L) = -2.596$  ;  $\theta_{p1} = 111.1^\circ$  or  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.596$  ;  $\theta_{p2} = 68.9^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 9.2^\circ$  ;  $\text{Im}(y_L) = -2.596 + (-3.049) = -5.645$ ;  $\theta_{p1} = 100.0^\circ$  ;  $\theta_{S1} = 160.8^\circ$  ;  
d2)  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.596 + (-3.049) = -0.454$ ;  $\theta_{p2} = 155.6^\circ$  ;  $\theta_{S1} = 160.8^\circ$  ;  
d3)  $\theta_{L1} = 9.2^\circ$  ;  $\text{Im}(y_L) = -2.596 + (3.049) = 0.454$ ;  $\theta_{p3} = 24.4^\circ$  ;  $\theta_{S2} = 14.0^\circ$  ;  
d4)  $\theta_{L2} = 46.8^\circ$  ;  $\text{Im}(y_L) = 2.596 + (3.049) = 5.645$ ;  $\theta_{p4} = 80.0^\circ$  ;  $\theta_{S2} = 14.0^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 9.2 + 160.8 = 169.9$  ;  $\theta_p = 100.0$  ;  $A \sim 17000.2$   
e2)  $\theta_s = 46.8 + 160.8 = 207.5$  ;  $\theta_p = 155.6$  ;  $A \sim 32293.4$   
e3)  $\theta_s = 9.2 + 14.0 = 23.2$  ;  $\theta_p = 24.4$  ;  $A \sim 565.6$   
e4)  $\theta_s = 46.8 + 14.0 = 60.8$  ;  $\theta_p = 80.0$  ;  $A \sim 4861.0$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 26

1.  $y = Y/Y_0 = Z_0/Z = 85\Omega / (33.8 - j \cdot 69.6)\Omega = 0.480 + j \cdot 0.988$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0352 - j \cdot 0.0178)] / (0.02 + 0.0352 - j \cdot 0.0178)$   
 $\Gamma = (-0.344) + j \cdot (0.212) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.404 \angle 148.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless ring coupler, matched at the input; Isolation  $I = 20.70\text{dB}$   
 $P_{\text{in}} = 3.80\text{mW} = 5.798\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.798\text{dBm} - 20.70\text{dB} = -14.90\text{dBm} = 32.343\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.465$ ,  $y_1 = 0.465$ ,  $y_2 = 0.885$ ,  $Z_1 = Z_0/y_1 = 107.5\Omega$ ,  $Z_2 = Z_0/y_2 = 56.5\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 27)\Omega} = 36.74\Omega$   
b)  $Z_L = 27\Omega$  parallel with  $0.43\text{pF}$  capacitor at  $9.3\text{GHz} = 18.49\Omega + j \cdot (-12.54)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (33.92)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 15.95\text{dB}$ ):  $G = G_1 + G_4 = 6.7 + 11.6 = 18.3\text{dB}$ ;  $G = G_2 + G_3 = 8.1 + 8.9 = 17.0\text{dB}$ ;  $G = G_2 + G_4 = 8.1 + 11.6 = 19.7\text{dB}$ ;  $G = G_3 + G_4 = 8.9 + 11.6 = 20.5\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.62\text{dB} = 1.153$ ,  $F_2 = 0.83\text{dB} = 1.211$ ,  $F_3 = 1.03\text{dB} = 1.268$ ,  $F_4 = 1.23\text{dB} = 1.327$ ,  $G_1 = 6.7\text{dB} = 4.677$ ,  $G_2 = 8.1\text{dB} = 6.457$ ;  $F(1,4) = 1.153 + (1.327 - 1)/4.677 = 1.223 = 0.88\text{dB}$ ;  $F(2,3) = 1.211 + (1.268 - 1)/6.457 = 1.261 = 1.01\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.614 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.136 > 1$  ;  $|\Delta| = |(-0.072) + j \cdot (0.264)| = 0.274 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 17.58 = 12.45\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.031$  ;  $C_1 = (-0.499) + j \cdot (0.084)$  ;  $\Gamma_S = (-0.808) + j \cdot (-0.136) = 0.819 \angle -170.5^\circ$   
 $B_2 = 0.818$  ;  $C_2 = (-0.229) + j \cdot (-0.324)$  ;  $\Gamma_L = (-0.448) + j \cdot (0.634) = 0.777 \angle 125.2^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 157.7^\circ$  ;  $\text{Im}(y_S) = -2.860$  ;  $\theta_{p1} = 109.3^\circ$  or  $\theta_{S2} = 12.7^\circ$  ;  $\text{Im}(y_S) = 2.860$  ;  $\theta_{p2} = 70.7^\circ$   
output:  $\theta_{L1} = 7.8^\circ$  ;  $\text{Im}(y_L) = -2.465$  ;  $\theta_{p1} = 112.1^\circ$  or  $\theta_{L2} = 46.9^\circ$  ;  $\text{Im}(y_L) = 2.465$  ;  $\theta_{p2} = 67.9^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 7.8^\circ$  ;  $\text{Im}(y_L) = -2.465 + (-2.860) = -5.324$ ;  $\theta_{p1} = 100.6^\circ$  ;  $\theta_{S1} = 157.7^\circ$  ;  
d2)  $\theta_{L2} = 46.9^\circ$  ;  $\text{Im}(y_L) = 2.465 + (-2.860) = -0.395$ ;  $\theta_{p2} = 158.4^\circ$  ;  $\theta_{S1} = 157.7^\circ$  ;  
d3)  $\theta_{L1} = 7.8^\circ$  ;  $\text{Im}(y_L) = -2.465 + (2.860) = 0.395$ ;  $\theta_{p3} = 21.6^\circ$  ;  $\theta_{S2} = 12.7^\circ$  ;  
d4)  $\theta_{L2} = 46.9^\circ$  ;  $\text{Im}(y_L) = 2.465 + (2.860) = 5.324$ ;  $\theta_{p4} = 79.4^\circ$  ;  $\theta_{S2} = 12.7^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma\theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 7.8 + 157.7 = 165.6$  ;  $\theta_p = 100.6$  ;  $A \sim 16665.1$   
e2)  $\theta_s = 46.9 + 157.7 = 204.7$  ;  $\theta_p = 158.4$  ;  $A \sim 32425.6$   
e3)  $\theta_s = 7.8 + 12.7 = 20.6$  ;  $\theta_p = 21.6$  ;  $A \sim 443.3$   
e4)  $\theta_s = 46.9 + 12.7 = 59.6$  ;  $\theta_p = 79.4$  ;  $A \sim 4731.7$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 27

1.  $y = Y/Y_0 = Z_0/Z = 70\Omega / (31.4 + j \cdot 55.9)\Omega = 0.535 - j \cdot 0.952$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0289 + j \cdot 0.0158)] / (0.02 + 0.0289 + j \cdot 0.0158)$   
 $\Gamma = (-0.259) + j \cdot (-0.239) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.353 \angle -137.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless ring coupler, matched at the input; Isolation  $I = D + C = 25.85\text{dB}$   
 $P_{\text{in}} = 4.00\text{mW} = 6.021\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 6.021\text{dBm} - 25.85\text{dB} = -19.83\text{dBm} = 10.401\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.516$ ,  $y_1 = 0.516$ ,  $y_2 = 0.857$ ,  $Z_1 = Z_0/y_1 = 96.9\Omega$ ,  $Z_2 = Z_0/y_2 = 58.4\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 57)\Omega} = 53.39\Omega$   
b)  $Z_L = 57\Omega$  parallel with  $0.79\text{pF}$  capacitor at  $6.6\text{GHz} = 12.70\Omega + j \cdot (-23.72)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (93.37)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 15.15\text{dB}$ ):  $G = G_1 + G_4 = 5.2 + 11.6 = 16.8\text{dB}$ ;  $G = G_2 + G_3 = 7.1 + 8.8 = 15.9\text{dB}$ ;  $G = G_2 + G_4 = 7.1 + 11.6 = 18.7\text{dB}$ ;  $G = G_3 + G_4 = 8.8 + 11.6 = 20.4\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.53\text{dB} = 1.130$ ,  $F_2 = 0.88\text{dB} = 1.225$ ,  $F_3 = 1.00\text{dB} = 1.259$ ,  $F_4 = 1.27\text{dB} = 1.340$ ,  $G_1 = 5.2\text{dB} = 3.311$ ,  $G_2 = 7.1\text{dB} = 5.129$ ;  $F(1,4) = 1.130 + (1.340 - 1)/3.311 = 1.232 = 0.91\text{dB}$ ;  $F(2,3) = 1.225 + (1.259 - 1)/5.129 = 1.291 = 1.11\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.632 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.106 > 1$  ;  $|\Delta| = |(-0.111) + j \cdot (-0.257)| = 0.280 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 20.24 = 13.06\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.051$  ;  $C_1 = (-0.518) + j \cdot (0.008)$  ;  $\Gamma_S = (-0.845) + j \cdot (-0.013) = 0.845 \angle -179.1^\circ$   
 $B_2 = 0.792$  ;  $C_2 = (-0.216) + j \cdot (-0.321)$  ;  $\Gamma_L = (-0.446) + j \cdot (0.663) = 0.799 \angle 123.9^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 163.4^\circ$  ;  $\text{Im}(y_S) = -3.164$  ;  $\theta_{p1} = 107.5^\circ$  or  $\theta_{S2} = 15.7^\circ$  ;  $\text{Im}(y_S) = 3.164$  ;  $\theta_{p2} = 72.5^\circ$   
output:  $\theta_{L1} = 9.6^\circ$  ;  $\text{Im}(y_L) = -2.657$  ;  $\theta_{p1} = 110.6^\circ$  or  $\theta_{L2} = 46.5^\circ$  ;  $\text{Im}(y_L) = 2.657$  ;  $\theta_{p2} = 69.4^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 9.6^\circ$  ;  $\text{Im}(y_L) = -2.657 + (-3.164) = -5.821$ ;  $\theta_{p1} = 99.7^\circ$  ;  $\theta_{S1} = 163.4^\circ$  ;  
d2)  $\theta_{L2} = 46.5^\circ$  ;  $\text{Im}(y_L) = 2.657 + (-3.164) = -0.507$ ;  $\theta_{p2} = 153.1^\circ$  ;  $\theta_{S1} = 163.4^\circ$  ;  
d3)  $\theta_{L1} = 9.6^\circ$  ;  $\text{Im}(y_L) = -2.657 + (3.164) = 0.507$ ;  $\theta_{p3} = 26.9^\circ$  ;  $\theta_{S2} = 15.7^\circ$  ;  
d4)  $\theta_{L2} = 46.5^\circ$  ;  $\text{Im}(y_L) = 2.657 + (3.164) = 5.821$ ;  $\theta_{p4} = 80.3^\circ$  ;  $\theta_{S2} = 15.7^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 9.6 + 163.4 = 173.0$  ;  $\theta_p = 99.7$  ;  $A \sim 17253.5$   
e2)  $\theta_s = 46.5 + 163.4 = 209.9$  ;  $\theta_p = 153.1$  ;  $A \sim 32146.3$   
e3)  $\theta_s = 9.6 + 15.7 = 25.3$  ;  $\theta_p = 26.9$  ;  $A \sim 679.2$   
e4)  $\theta_s = 46.5 + 15.7 = 62.2$  ;  $\theta_p = 80.3$  ;  $A \sim 4994.5$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 28

1.  $y = Y/Y_0 = Z_0/Z = 70\Omega / (31.9 - j \cdot 49.7)\Omega = 0.640 + j \cdot 0.998$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0260 + j \cdot 0.0291)] / (0.02 + 0.0260 + j \cdot 0.0291)$   
 $\Gamma = (-0.379) + j \cdot (-0.393) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.546 \angle -134.0^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless ring coupler, matched at the input; Isolation  $I = D + C = 27.95\text{dB}$   
 $P_{\text{in}} = 3.25\text{mW} = 5.119\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.119\text{dBm} - 27.95\text{dB} = -22.83\text{dBm} = 5.211\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.606$ ,  $y_1 = 0.606$ ,  $y_2 = 0.795$ ,  $Z_1 = Z_0/y_1 = 82.5\Omega$ ,  $Z_2 = Z_0/y_2 = 62.9\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 51)\Omega} = 50.50\Omega$   
b)  $Z_L = 51\Omega$  parallel with  $1.1\text{nH}$  inductor at  $7.4\text{GHz} = 25.80\Omega + j \cdot (25.50)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (-49.41)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 14.50\text{dB}$ ):  $G = G_1 + G_4 = 5.8 + 10.3 = 16.1\text{dB}$ ;  $G = G_2 + G_3 = 7.7 + 8.6 = 16.3\text{dB}$ ;  $G = G_2 + G_4 = 7.7 + 10.3 = 18.0\text{dB}$ ;  $G = G_3 + G_4 = 8.6 + 10.3 = 18.9\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.66\text{dB} = 1.164$ ,  $F_2 = 0.87\text{dB} = 1.222$ ,  $F_3 = 1.05\text{dB} = 1.274$ ,  $F_4 = 1.23\text{dB} = 1.327$ ,  $G_1 = 5.8\text{dB} = 3.802$ ,  $G_2 = 7.7\text{dB} = 5.888$ ;  $F(1,4) = 1.164 + (1.327 - 1)/3.802 = 1.250 = 0.97\text{dB}$ ;  $F(2,3) = 1.222 + (1.274 - 1)/5.888 = 1.277 = 1.06\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.632 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.223 > 1$  ;  $|\Delta| = |(0.261) + j \cdot (0.003)| = 0.261 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 9.59 = 9.82\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.029$  ;  $C_1 = (-0.311) + j \cdot (0.396)$  ;  $\Gamma_S = (-0.502) + j \cdot (-0.639) = 0.813 \angle -128.1^\circ$   
 $B_2 = 0.835$  ;  $C_2 = (-0.393) + j \cdot (-0.094)$  ;  $\Gamma_L = (-0.752) + j \cdot (0.181) = 0.773 \angle 166.5^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 136.2^\circ$  ;  $\text{Im}(y_S) = -2.789$  ;  $\theta_{p1} = 109.7^\circ$  or  $\theta_{S2} = 171.9^\circ$  ;  $\text{Im}(y_S) = 2.789$  ;  $\theta_{p2} = 70.3^\circ$   
output:  $\theta_{L1} = 167.1^\circ$  ;  $\text{Im}(y_L) = -2.436$  ;  $\theta_{p1} = 112.3^\circ$  or  $\theta_{L2} = 26.4^\circ$  ;  $\text{Im}(y_L) = 2.436$  ;  $\theta_{p2} = 67.7^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 167.1^\circ$  ;  $\text{Im}(y_L) = -2.436 + (-2.789) = -5.225$ ;  $\theta_{p1} = 100.8^\circ$  ;  $\theta_{S1} = 136.2^\circ$  ;  
d2)  $\theta_{L2} = 26.4^\circ$  ;  $\text{Im}(y_L) = 2.436 + (-2.789) = -0.352$ ;  $\theta_{p2} = 160.6^\circ$  ;  $\theta_{S1} = 136.2^\circ$  ;  
d3)  $\theta_{L1} = 167.1^\circ$  ;  $\text{Im}(y_L) = -2.436 + (2.789) = 0.352$ ;  $\theta_{p3} = 19.4^\circ$  ;  $\theta_{S2} = 171.9^\circ$  ;  
d4)  $\theta_{L2} = 26.4^\circ$  ;  $\text{Im}(y_L) = 2.436 + (2.789) = 5.225$ ;  $\theta_{p4} = 79.2^\circ$  ;  $\theta_{S2} = 171.9^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 167.1 + 136.2 = 303.3$  ;  $\theta_p = 100.8$  ;  $A \sim 30583.6$   
e2)  $\theta_s = 26.4 + 136.2 = 162.7$  ;  $\theta_p = 160.6$  ;  $A \sim 26124.9$   
e3)  $\theta_s = 167.1 + 171.9 = 339.0$  ;  $\theta_p = 19.4$  ;  $A \sim 6581.3$   
e4)  $\theta_s = 26.4 + 171.9 = 198.3$  ;  $\theta_p = 79.2$  ;  $A \sim 15701.2$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 29

1.  $y = Y/Y_0 = Z_0/Z = 60\Omega / (49.2 - j \cdot 38.3)\Omega = 0.759 + j \cdot 0.591$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0305 + j \cdot 0.0172)] / (0.02 + 0.0305 + j \cdot 0.0172)$   
 $\Gamma = (-0.290) + j \cdot (-0.242) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.378 \angle -140.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless ring coupler, matched at the input; Isolation  $I = D + C = 27.85\text{dB}$

$P_{\text{in}} = 1.15\text{mW} = 0.607\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 0.607\text{dBm} - 27.85\text{dB} = -27.24\text{dBm} = 1.887\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.516$ ,  $y_1 = 0.516$ ,  $y_2 = 0.857$ ,  $Z_1 = Z_0/y_1 = 96.9\Omega$ ,  $Z_2 = Z_0/y_2 = 58.4\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 53)\Omega} = 51.48\Omega$

b)  $Z_L = 53\Omega$  parallel with  $0.40\text{pF}$  capacitor at  $8.0\text{GHz} = 24.82\Omega + j \cdot (-26.45)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (53.28)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 14.45\text{dB}$ ):  $G = G_1 + G_4 = 5.2 + 11.2 = 16.4\text{dB}$ ;  $G = G_2 + G_3 = 8.2 + 8.3 = 16.5\text{dB}$ ;  $G = G_2 + G_4 = 8.2 + 11.2 = 19.4\text{dB}$ ;  $G = G_3 + G_4 = 8.3 + 11.2 = 19.5\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.57\text{dB} = 1.140$ ,  $F_2 = 0.73\text{dB} = 1.183$ ,  $F_3 = 1.08\text{dB} = 1.282$ ,  $F_4 = 1.15\text{dB} = 1.303$ ,  $G_1 = 5.2\text{dB} = 3.311$ ,  $G_2 = 8.2\text{dB} = 6.607$ ;  $F(1,4) = 1.140 + (1.303 - 1)/3.311 = 1.232 = 0.91\text{dB}$ ;  $F(2,3) = 1.183 + (1.282 - 1)/6.607 = 1.229 = 0.90\text{dB}$ ;

$F(1,4) > F(2,3) \rightarrow$  For minimal noise factor we must use devices 2,3

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.640 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.189 > 1$  ;  $|\Delta| = |(0.262) + j \cdot (0.095)| = 0.279 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 10.36 = 10.15\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.029$  ;  $C_1 = (-0.373) + j \cdot (0.340)$  ;  $\Gamma_S = (-0.608) + j \cdot (-0.555) = 0.823 \angle -137.6^\circ$

$B_2 = 0.815$  ;  $C_2 = (-0.374) + j \cdot (-0.129)$  ;  $\Gamma_L = (-0.738) + j \cdot (0.254) = 0.780 \angle 161.0^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 141.5^\circ$  ;  $\text{Im}(y_S) = -2.897$  ;  $\theta_{p1} = 109.0^\circ$  or  $\theta_{S2} = 176.1^\circ$  ;  $\text{Im}(y_S) = 2.897$  ;  $\theta_{p2} = 71.0^\circ$

output:  $\theta_{L1} = 170.1^\circ$  ;  $\text{Im}(y_L) = -2.496$  ;  $\theta_{p1} = 111.8^\circ$  or  $\theta_{L2} = 28.9^\circ$  ;  $\text{Im}(y_L) = 2.496$  ;  $\theta_{p2} = 68.2^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 170.1^\circ$  ;  $\text{Im}(y_L) = -2.496 + (-2.897) = -5.393$ ;  $\theta_{p1} = 100.5^\circ$  ;  $\theta_{S1} = 141.5^\circ$  ;

d2)  $\theta_{L2} = 28.9^\circ$  ;  $\text{Im}(y_L) = 2.496 + (-2.897) = -0.401$ ;  $\theta_{p2} = 158.1^\circ$  ;  $\theta_{S1} = 141.5^\circ$  ;

d3)  $\theta_{L1} = 170.1^\circ$  ;  $\text{Im}(y_L) = -2.496 + (2.897) = 0.401$ ;  $\theta_{p3} = 21.9^\circ$  ;  $\theta_{S2} = 176.1^\circ$  ;

d4)  $\theta_{L2} = 28.9^\circ$  ;  $\text{Im}(y_L) = 2.496 + (2.897) = 5.393$ ;  $\theta_{p4} = 79.5^\circ$  ;  $\theta_{S2} = 176.1^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 170.1 + 141.5 = 311.7$  ;  $\theta_p = 100.5$  ;  $A \sim 31322.5$

e2)  $\theta_s = 28.9 + 141.5 = 170.4$  ;  $\theta_p = 158.1$  ;  $A \sim 26941.0$

e3)  $\theta_s = 170.1 + 176.1 = 346.3$  ;  $\theta_p = 21.9$  ;  $A \sim 7568.0$

e4)  $\theta_s = 28.9 + 176.1 = 205.0$  ;  $\theta_p = 79.5$  ;  $A \sim 16294.6$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 30

1.  $y = Y/Y_0 = Z_0/Z = 85\Omega / (35.7 + j \cdot 66.3)\Omega = 0.535 - j \cdot 0.994$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0133 - j \cdot 0.0223)] / (0.02 + 0.0133 - j \cdot 0.0223)$   
 $\Gamma = (-0.171) + j \cdot (0.555) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.581 \angle 107.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 21.50\text{dB}$   
 $P_{\text{in}} = 1.80\text{mW} = 2.553\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 2.553\text{dBm} - 21.50\text{dB} = -18.95\text{dBm} = 12.743\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.617$ ,  $Z_{\text{CE}} = 102.67\Omega$ ,  $Z_{\text{CO}} = 24.35\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 46)\Omega} = 47.96\Omega$   
b)  $Z_L = 46\Omega$  series with  $0.32\text{pF}$  capacitor at  $7.3\text{GHz} = 46.00\Omega + j \cdot (-68.13)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 15.66\Omega + j \cdot (23.19)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 15.80\text{dB}$ ):  $G = G_1 + G_4 = 5.5 + 11.9 = 17.4\text{dB}$ ;  $G = G_2 + G_3 = 7.5 + 9.8 = 17.3\text{dB}$ ;  $G = G_2 + G_4 = 7.5 + 11.9 = 19.4\text{dB}$ ;  $G = G_3 + G_4 = 9.8 + 11.9 = 21.7\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.64\text{dB} = 1.159$ ,  $F_2 = 0.84\text{dB} = 1.213$ ,  $F_3 = 0.95\text{dB} = 1.245$ ,  $F_4 = 1.25\text{dB} = 1.334$ ,  $G_1 = 5.5\text{dB} = 3.548$ ,  $G_2 = 7.5\text{dB} = 5.623$ ;  $F(1,4) = 1.159 + (1.334 - 1)/3.548 = 1.253 = 0.98\text{dB}$ ;  $F(2,3) = 1.213 + (1.245 - 1)/5.623 = 1.273 = 1.05\text{dB}$ ;  
 $F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.626 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.110 > 1$  ;  $|\Delta| = |(-0.103) + j \cdot (0.261)| = 0.280 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 19.36 = 12.87\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.043$  ;  $C_1 = (-0.512) + j \cdot (0.034)$  ;  $\Gamma_S = (-0.837) + j \cdot (-0.055) = 0.839 \angle -176.2^\circ$   
 $B_2 = 0.800$  ;  $C_2 = (-0.218) + j \cdot (-0.323)$  ;  $\Gamma_L = (-0.444) + j \cdot (0.659) = 0.795 \angle 124.0^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 161.6^\circ$  ;  $\text{Im}(y_S) = -3.088$  ;  $\theta_{p1} = 107.9^\circ$  or  $\theta_{S2} = 14.6^\circ$  ;  $\text{Im}(y_S) = 3.088$  ;  $\theta_{p2} = 72.1^\circ$   
output:  $\theta_{L1} = 9.3^\circ$  ;  $\text{Im}(y_L) = -2.617$  ;  $\theta_{p1} = 110.9^\circ$  or  $\theta_{L2} = 46.7^\circ$  ;  $\text{Im}(y_L) = 2.617$  ;  $\theta_{p2} = 69.1^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 9.3^\circ$  ;  $\text{Im}(y_L) = -2.617 + (-3.088) = -5.705$ ;  $\theta_{p1} = 99.9^\circ$  ;  $\theta_{S1} = 161.6^\circ$  ;  
d2)  $\theta_{L2} = 46.7^\circ$  ;  $\text{Im}(y_L) = 2.617 + (-3.088) = -0.471$ ;  $\theta_{p2} = 154.8^\circ$  ;  $\theta_{S1} = 161.6^\circ$  ;  
d3)  $\theta_{L1} = 9.3^\circ$  ;  $\text{Im}(y_L) = -2.617 + (3.088) = 0.471$ ;  $\theta_{p3} = 25.2^\circ$  ;  $\theta_{S2} = 14.6^\circ$  ;  
d4)  $\theta_{L2} = 46.7^\circ$  ;  $\text{Im}(y_L) = 2.617 + (3.088) = 5.705$ ;  $\theta_{p4} = 80.1^\circ$  ;  $\theta_{S2} = 14.6^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 9.3 + 161.6 = 170.9$  ;  $\theta_p = 99.9$  ;  $A \sim 17084.9$   
e2)  $\theta_s = 46.7 + 161.6 = 208.3$  ;  $\theta_p = 154.8$  ;  $A \sim 32245.7$   
e3)  $\theta_s = 9.3 + 14.6 = 23.9$  ;  $\theta_p = 25.2$  ;  $A \sim 602.3$   
e4)  $\theta_s = 46.7 + 14.6 = 61.3$  ;  $\theta_p = 80.1$  ;  $A \sim 4905.0$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 31

1.  $y = Y/Y_0 = Z_0/Z = 100\Omega / (51.9 + j \cdot 67.1)\Omega = 0.721 - j \cdot 0.932$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0114 - j \cdot 0.0232)] / (0.02 + 0.0114 - j \cdot 0.0232)$   
 $\Gamma = (-0.176) + j \cdot (0.609) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.634 \angle 106.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless quadrature coupler, matched at the input; Isolation  $I = D + C = 30.70\text{dB}$

$P_{\text{in}} = 3.40\text{mW} = 5.315\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 5.315\text{dBm} - 30.70\text{dB} = -25.39\text{dBm} = 2.894\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.484$ ,  $y_2 = 1.143$ ,  $y_1 = 0.553$ ,  $Z_1 = Z_0/y_1 = 90.4\Omega$ ,  $Z_2 = Z_0/y_2 = 43.7\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 41)\Omega} = 45.28\Omega$

b)  $Z_L = 41\Omega$  series with  $0.74\text{nH}$  inductor at  $9.5\text{GHz} = 41.00\Omega + j \cdot (44.17)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 23.14\Omega + j \cdot (-24.93)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.70\text{dB}$ ):  $G = G_1 + G_4 = 6.8 + 11.8 = 18.6\text{dB}$ ;  $G = G_2 + G_3 = 7.4 + 9.8 = 17.2\text{dB}$ ;  $G = G_2 + G_4 = 7.4 + 11.8 = 19.2\text{dB}$ ;  $G = G_3 + G_4 = 9.8 + 11.8 = 21.6\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.63\text{dB} = 1.156$ ,  $F_2 = 0.88\text{dB} = 1.225$ ,  $F_3 = 0.99\text{dB} = 1.256$ ,  $F_4 = 1.26\text{dB} = 1.337$ ,  $G_1 = 6.8\text{dB} = 4.786$ ,  $G_2 = 7.4\text{dB} = 5.495$ ;  $F(1,4) = 1.156 + (1.337 - 1)/4.786 = 1.226 = 0.89\text{dB}$ ;  $F(2,3) = 1.225 + (1.256 - 1)/5.495 = 1.286 = 1.09\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.640 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.182 > 1$  ;  $|\Delta| = |(0.246) + j \cdot (0.141)| = 0.284 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{K^2 - 1}] = 10.62 = 10.26\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.026$  ;  $C_1 = (-0.400) + j \cdot (0.307)$  ;  $\Gamma_S = (-0.653) + j \cdot (-0.502) = 0.824 \angle -142.5^\circ$

$B_2 = 0.812$  ;  $C_2 = (-0.364) + j \cdot (-0.151)$  ;  $\Gamma_L = (-0.722) + j \cdot (0.299) = 0.781 \angle 157.5^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 144.0^\circ$  ;  $\text{Im}(y_S) = -2.906$  ;  $\theta_{p1} = 109.0^\circ$  or  $\theta_{S2} = 178.5^\circ$  ;  $\text{Im}(y_S) = 2.906$  ;  $\theta_{p2} = 71.0^\circ$

output:  $\theta_{L1} = 171.9^\circ$  ;  $\text{Im}(y_L) = -2.503$  ;  $\theta_{p1} = 111.8^\circ$  or  $\theta_{L2} = 30.6^\circ$  ;  $\text{Im}(y_L) = 2.503$  ;  $\theta_{p2} = 68.2^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 171.9^\circ$  ;  $\text{Im}(y_L) = -2.503 + (-2.906) = -5.409$ ;  $\theta_{p1} = 100.5^\circ$  ;  $\theta_{S1} = 144.0^\circ$  ;

d2)  $\theta_{L2} = 30.6^\circ$  ;  $\text{Im}(y_L) = 2.503 + (-2.906) = -0.403$ ;  $\theta_{p2} = 158.1^\circ$  ;  $\theta_{S1} = 144.0^\circ$  ;

d3)  $\theta_{L1} = 171.9^\circ$  ;  $\text{Im}(y_L) = -2.503 + (2.906) = 0.403$ ;  $\theta_{p3} = 21.9^\circ$  ;  $\theta_{S2} = 178.5^\circ$  ;

d4)  $\theta_{L2} = 30.6^\circ$  ;  $\text{Im}(y_L) = 2.503 + (2.906) = 5.409$ ;  $\theta_{p4} = 79.5^\circ$  ;  $\theta_{S2} = 178.5^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 171.9 + 144.0 = 315.9$  ;  $\theta_p = 100.5$  ;  $A \sim 31740.9$

e2)  $\theta_s = 30.6 + 144.0 = 174.5$  ;  $\theta_p = 158.1$  ;  $A \sim 27585.9$

e3)  $\theta_s = 171.9 + 178.5 = 350.4$  ;  $\theta_p = 21.9$  ;  $A \sim 7691.5$

e4)  $\theta_s = 30.6 + 178.5 = 209.1$  ;  $\theta_p = 79.5$  ;  $A \sim 16626.7$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 32

1.  $y = Y/Y_0 = Z_0/Z = 60\Omega / (62.2 - j \cdot 58.7)\Omega = 0.510 + j \cdot 0.482$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0299 + j \cdot 0.0282)] / (0.02 + 0.0299 + j \cdot 0.0282)$   
 $\Gamma = (-0.392) + j \cdot (-0.343) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.521 \angle -138.8^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = D + C = 29.00\text{dB}$

$P_{\text{in}} = 1.95\text{mW} = 2.900\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 2.900\text{dBm} - 29.00\text{dB} = -26.10\text{dBm} = 2.455\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.473$ ,  $Z_{\text{CE}} = 83.61\Omega$ ,  $Z_{\text{CO}} = 29.90\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 53)\Omega} = 51.48\Omega$

b)  $Z_L = 53\Omega$  parallel with  $0.35\text{pF}$  capacitor at  $7.1\text{GHz} = 31.46\Omega + j \cdot (-26.03)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (41.38)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.10\text{dB}$ ):  $G = G_1 + G_4 = 6.2 + 11.5 = 17.7\text{dB}$ ;  $G = G_2 + G_3 = 7.6 + 8.2 = 15.8\text{dB}$ ;  $G = G_2 + G_4 = 7.6 + 11.5 = 19.1\text{dB}$ ;  $G = G_3 + G_4 = 8.2 + 11.5 = 19.7\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.52\text{dB} = 1.127$ ,  $F_2 = 0.89\text{dB} = 1.227$ ,  $F_3 = 0.98\text{dB} = 1.253$ ,  $F_4 = 1.21\text{dB} = 1.321$ ,  $G_1 = 6.2\text{dB} = 4.169$ ,  $G_2 = 7.6\text{dB} = 5.754$ ;  $F(1,4) = 1.127 + (1.321 - 1)/4.169 = 1.204 = 0.81\text{dB}$ ;  $F(2,3) = 1.227 + (1.253 - 1)/5.754 = 1.283 = 1.08\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.638 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.197 > 1$  ;  $|\Delta| = |(0.265) + j \cdot (0.072)| = 0.274 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 10.18 = 10.08\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.029$  ;  $C_1 = (-0.358) + j \cdot (0.355)$  ;  $\Gamma_S = (-0.583) + j \cdot (-0.578) = 0.821 \angle -135.2^\circ$

$B_2 = 0.820$  ;  $C_2 = (-0.379) + j \cdot (-0.120)$  ;  $\Gamma_L = (-0.742) + j \cdot (0.236) = 0.779 \angle 162.4^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 140.2^\circ$  ;  $\text{Im}(y_S) = -2.872$  ;  $\theta_{p1} = 109.2^\circ$  or  $\theta_{S2} = 175.0^\circ$  ;  $\text{Im}(y_S) = 2.872$  ;  $\theta_{p2} = 70.8^\circ$

output:  $\theta_{L1} = 169.4^\circ$  ;  $\text{Im}(y_L) = -2.483$  ;  $\theta_{p1} = 111.9^\circ$  or  $\theta_{L2} = 28.2^\circ$  ;  $\text{Im}(y_L) = 2.483$  ;  $\theta_{p2} = 68.1^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 169.4^\circ$  ;  $\text{Im}(y_L) = -2.483 + (-2.872) = -5.355$ ;  $\theta_{p1} = 100.6^\circ$  ;  $\theta_{S1} = 140.2^\circ$  ;

d2)  $\theta_{L2} = 28.2^\circ$  ;  $\text{Im}(y_L) = 2.483 + (-2.872) = -0.389$ ;  $\theta_{p2} = 158.8^\circ$  ;  $\theta_{S1} = 140.2^\circ$  ;

d3)  $\theta_{L1} = 169.4^\circ$  ;  $\text{Im}(y_L) = -2.483 + (2.872) = 0.389$ ;  $\theta_{p3} = 21.2^\circ$  ;  $\theta_{S2} = 175.0^\circ$  ;

d4)  $\theta_{L2} = 28.2^\circ$  ;  $\text{Im}(y_L) = 2.483 + (2.872) = 5.355$ ;  $\theta_{p4} = 79.4^\circ$  ;  $\theta_{S2} = 175.0^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 169.4 + 140.2 = 309.6$  ;  $\theta_p = 100.6$  ;  $A \sim 31137.7$

e2)  $\theta_s = 28.2 + 140.2 = 168.4$  ;  $\theta_p = 158.8$  ;  $A \sim 26740.5$

e3)  $\theta_s = 169.4 + 175.0 = 344.4$  ;  $\theta_p = 21.2$  ;  $A \sim 7317.3$

e4)  $\theta_s = 28.2 + 175.0 = 203.3$  ;  $\theta_p = 79.4$  ;  $A \sim 16145.9$

Smallest substrate area is occupied by solution e3 (d3)



### Subject no. 33

1.  $y = Y/Y_0 = Z_0/Z = 70\Omega / (34.3 - j \cdot 38.7)\Omega = 0.898 + j \cdot 1.013$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0388 + j \cdot 0.0238)] / (0.02 + 0.0388 + j \cdot 0.0238)$   
 $\Gamma = (-0.415) + j \cdot (-0.237) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.478 \angle -150.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless coupled line coupler, matched at the input; Isolation  $I = 24.60\text{dB}$

$P_{\text{in}} = 3.05\text{mW} = 4.843\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 4.843\text{dBm} - 24.60\text{dB} = -19.76\text{dBm} = 10.575\mu\text{W}$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.606$ ,  $Z_{\text{CE}} = 100.95\Omega$ ,  $Z_{\text{CO}} = 24.76\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 31)\Omega} = 39.37\Omega$

b)  $Z_L = 31\Omega$  parallel with  $0.62\text{nH}$  inductor at  $9.0\text{GHz} = 17.40\Omega + j \cdot (15.38)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (-44.21)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 16.70\text{dB}$ ):  $G = G_1 + G_4 = 6.0 + 11.1 = 17.1\text{dB}$ ;  $G = G_2 + G_3 = 8.1 + 9.9 = 18.0\text{dB}$ ;  $G = G_2 + G_4 = 8.1 + 11.1 = 19.2\text{dB}$ ;  $G = G_3 + G_4 = 9.9 + 11.1 = 21.0\text{dB}$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.53\text{dB} = 1.130$ ,  $F_2 = 0.77\text{dB} = 1.194$ ,  $F_3 = 0.92\text{dB} = 1.236$ ,  $F_4 = 1.27\text{dB} = 1.340$ ,  $G_1 = 6.0\text{dB} = 3.981$ ,  $G_2 = 8.1\text{dB} = 6.457$ ;  $F(1,4) = 1.130 + (1.340 - 1)/3.981 = 1.215 = 0.85\text{dB}$ ;  $F(2,3) = 1.194 + (1.236 - 1)/6.457 = 1.247 = 0.96\text{dB}$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.638 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.103 > 1$  ;  $|\Delta| = |(-0.118) + j \cdot (0.254)| = 0.280 < 1$

b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 21.15 = 13.25\text{dB}$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.058$  ;  $C_1 = (-0.522) + j \cdot (-0.018)$  ;  $\Gamma_S = (-0.850) + j \cdot (0.030) = 0.851 \angle 178.0^\circ$

$B_2 = 0.785$  ;  $C_2 = (-0.213) + j \cdot (-0.318)$  ;  $\Gamma_L = (-0.447) + j \cdot (0.667) = 0.803 \angle 123.8^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 165.2^\circ$  ;  $\text{Im}(y_S) = -3.239$  ;  $\theta_{p1} = 107.2^\circ$  or  $\theta_{S2} = 16.9^\circ$  ;  $\text{Im}(y_S) = 3.239$  ;  $\theta_{p2} = 72.8^\circ$

output:  $\theta_{L1} = 9.8^\circ$  ;  $\text{Im}(y_L) = -2.695$  ;  $\theta_{p1} = 110.4^\circ$  or  $\theta_{L2} = 46.4^\circ$  ;  $\text{Im}(y_L) = 2.695$  ;  $\theta_{p2} = 69.6^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 9.8^\circ$  ;  $\text{Im}(y_L) = -2.695 + (-3.239) = -5.934$ ;  $\theta_{p1} = 99.6^\circ$  ;  $\theta_{S1} = 165.2^\circ$  ;

d2)  $\theta_{L2} = 46.4^\circ$  ;  $\text{Im}(y_L) = 2.695 + (-3.239) = -0.544$ ;  $\theta_{p2} = 151.5^\circ$  ;  $\theta_{S1} = 165.2^\circ$  ;

d3)  $\theta_{L1} = 9.8^\circ$  ;  $\text{Im}(y_L) = -2.695 + (3.239) = 0.544$ ;  $\theta_{p3} = 28.5^\circ$  ;  $\theta_{S2} = 16.9^\circ$  ;

d4)  $\theta_{L2} = 46.4^\circ$  ;  $\text{Im}(y_L) = 2.695 + (3.239) = 5.934$ ;  $\theta_{p4} = 80.4^\circ$  ;  $\theta_{S2} = 16.9^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .

e1)  $\theta_s = 9.8 + 165.2 = 175.0$  ;  $\theta_p = 99.6$  ;  $A \sim 17420.8$

e2)  $\theta_s = 46.4 + 165.2 = 211.5$  ;  $\theta_p = 151.5$  ;  $A \sim 32042.9$

e3)  $\theta_s = 9.8 + 16.9 = 26.7$  ;  $\theta_p = 28.5$  ;  $A \sim 760.7$

e4)  $\theta_s = 46.4 + 16.9 = 63.2$  ;  $\theta_p = 80.4$  ;  $A \sim 5086.7$

Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 34

1.  $y = Y/Y_0 = Z_0/Z = 80\Omega / (33.0 - j \cdot 54.7)\Omega = 0.647 + j \cdot 1.072$
2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0206 - j \cdot 0.0147)] / (0.02 + 0.0206 - j \cdot 0.0147)$   
 $\Gamma = (-0.129) + j \cdot (0.315) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.341 \angle 112.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$
3. a) Lossless quadrature coupler, matched at the input; Isolation  $I = D + C = 30.30\text{dB}$   
 $P_{\text{in}} = 1.55\text{mW} = 1.903\text{dBm}$ ;  $P_{\text{is}} = P_{\text{in}} - I = 1.903\text{dBm} - 30.30\text{dB} = -28.40\text{dBm} = 1.447\mu\text{W}$   
b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.457$ ,  $y_2 = 1.124$ ,  $y_1 = 0.514$ ,  $Z_1 = Z_0/y_1 = 97.3\Omega$ ,  $Z_2 = Z_0/y_2 = 44.5\Omega$
4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 58)\Omega} = 53.85\Omega$   
b)  $Z_L = 58\Omega$  series with  $0.33\text{pF}$  capacitor at  $7.6\text{GHz} = 58.00\Omega + j \cdot (-63.46)\Omega$   
 $\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{\text{in}} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 22.76\Omega + j \cdot (24.90)\Omega$
5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).  
Valid combinations ( $G > 16.65\text{dB}$ ):  $G = G_1 + G_4 = 6.7 + 10.2 = 16.9\text{dB}$ ;  $G = G_2 + G_3 = 8.4 + 9.8 = 18.2\text{dB}$ ;  $G = G_2 + G_4 = 8.4 + 10.2 = 18.6\text{dB}$ ;  $G = G_3 + G_4 = 9.8 + 10.2 = 20.0\text{dB}$ ;  
b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;  
From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$   
 $F_1 = 0.69\text{dB} = 1.172$ ,  $F_2 = 0.77\text{dB} = 1.194$ ,  $F_3 = 1.07\text{dB} = 1.279$ ,  $F_4 = 1.29\text{dB} = 1.346$ ,  $G_1 = 6.7\text{dB} = 4.677$ ,  
 $G_2 = 8.4\text{dB} = 6.918$ ;  $F(1,4) = 1.172 + (1.346 - 1)/4.677 = 1.246 = 0.96\text{dB}$ ;  $F(2,3) = 1.194 + (1.279 - 1)/6.918 = 1.244 = 0.95\text{dB}$ ;  
 $F(1,4) > F(2,3) \rightarrow$  For minimal noise factor we must use devices 2,3
6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:  
 $|S_{11}| = 0.650 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.099 > 1$  ;  $|\Delta| = |(-0.133) + j \cdot (0.247)| = 0.281 < 1$   
b\_1)  $G_{\text{Tmax}} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 23.05 = 13.63\text{dB}$   
b\_2) Complex calculus from C8/2017, S106:  
 $B_1 = 1.073$  ;  $C_1 = (-0.526) + j \cdot (-0.072)$  ;  $\Gamma_S = (-0.853) + j \cdot (0.117) = 0.861 \angle 172.2^\circ$   
 $B_2 = 0.769$  ;  $C_2 = (-0.208) + j \cdot (-0.313)$  ;  $\Gamma_L = (-0.448) + j \cdot (0.675) = 0.810 \angle 123.6^\circ$   
c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines  
input:  $\theta_{S1} = 168.6^\circ$  ;  $\text{Im}(y_S) = -3.381$  ;  $\theta_{p1} = 106.5^\circ$  or  $\theta_{S2} = 19.2^\circ$  ;  $\text{Im}(y_S) = 3.381$  ;  $\theta_{p2} = 73.5^\circ$   
output:  $\theta_{L1} = 10.3^\circ$  ;  $\text{Im}(y_L) = -2.761$  ;  $\theta_{p1} = 109.9^\circ$  or  $\theta_{L2} = 46.2^\circ$  ;  $\text{Im}(y_L) = 2.761$  ;  $\theta_{p2} = 70.1^\circ$   
d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):  
d1)  $\theta_{L1} = 10.3^\circ$  ;  $\text{Im}(y_L) = -2.761 + (-3.381) = -6.142$ ;  $\theta_{p1} = 99.2^\circ$  ;  $\theta_{S1} = 168.6^\circ$  ;  
d2)  $\theta_{L2} = 46.2^\circ$  ;  $\text{Im}(y_L) = 2.761 + (-3.381) = -0.620$ ;  $\theta_{p2} = 148.2^\circ$  ;  $\theta_{S1} = 168.6^\circ$  ;  
d3)  $\theta_{L1} = 10.3^\circ$  ;  $\text{Im}(y_L) = -2.761 + (3.381) = 0.620$ ;  $\theta_{p3} = 31.8^\circ$  ;  $\theta_{S2} = 19.2^\circ$  ;  
d4)  $\theta_{L2} = 46.2^\circ$  ;  $\text{Im}(y_L) = 2.761 + (3.381) = 6.142$ ;  $\theta_{p4} = 80.8^\circ$  ;  $\theta_{S2} = 19.2^\circ$  ;  
e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}} \sim \text{Substrate Area}$ . We must compute all solutions for d) and compare individual products  $\Sigma \theta_{\text{serie}} \times \theta_{\text{paralel}}$ .  
e1)  $\theta_s = 10.3 + 168.6 = 178.9$  ;  $\theta_p = 99.2$  ;  $A \sim 17750.9$   
e2)  $\theta_s = 46.2 + 168.6 = 214.8$  ;  $\theta_p = 148.2$  ;  $A \sim 31832.7$   
e3)  $\theta_s = 10.3 + 19.2 = 29.5$  ;  $\theta_p = 31.8$  ;  $A \sim 936.5$   
e4)  $\theta_s = 46.2 + 19.2 = 65.4$  ;  $\theta_p = 80.8$  ;  $A \sim 5279.7$   
Smallest substrate area is occupied by solution e3 (d3)

## Subject no. 35

1.  $y = Y/Y_0 = Z_0/Z = 100\Omega / (55.9 + j \cdot 61.3)\Omega = 0.812 - j \cdot 0.891$

2.  $Y_0 = 0.02S$ ;  $\Gamma = (Y_0 - Y) / (Y_0 + Y) = [0.02 - (0.0275 - j \cdot 0.0313)] / (0.02 + 0.0275 - j \cdot 0.0313)$   
 $\Gamma = (-0.413) + j \cdot (0.387) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.566 \angle 136.9^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. a) Lossless quadrature coupler, matched at the input; Isolation I = 20.20dB

$P_{in} = 2.50mW = 3.979dBm$ ;  $P_{is} = P_{in} - I = 3.979dBm - 20.20dB = -16.22dBm = 23.875\mu W$

b) L2, C12/2017,  $\beta = 10^{-C/20} = 0.585$ ,  $y_2 = 1.234$ ,  $y_1 = 0.722$ ,  $Z_1 = Z_0/y_1 = 69.2 \Omega$ ,  $Z_2 = Z_0/y_2 = 40.5\Omega$

4. a)  $Z_1 = \sqrt{(Z_0 \cdot R_L)} = \sqrt{(50 \cdot 45)\Omega} = 47.43\Omega$

b)  $Z_L = 45\Omega$  parallel with 0.73nH inductor at 7.6GHz =  $16.88\Omega + j \cdot (21.79)\Omega$

$\theta = \pi/4$ ,  $\tan(\beta \cdot l) \rightarrow \infty$ ,  $Z_{in} = Z_1^2/Z_L = Z_0 \cdot R_L/Z_L = 50.00\Omega + j \cdot (-64.55)\Omega$

5. a) From the 6 possible combinations, 2 don't meet the required gain (1,2 ; 1,3).

Valid combinations ( $G > 15.40dB$ ):  $G = G_1 + G_4 = 6.8 + 10.5 = 17.3dB$ ;  $G = G_2 + G_3 = 7.6 + 8.2 = 15.8dB$ ;  $G = G_2 + G_4 = 7.6 + 10.5 = 18.1dB$ ;  $G = G_3 + G_4 = 8.2 + 10.5 = 18.7dB$ ;

b) Friis Formula (C9/2017, S92),  $F = F_a + (F_b - 1)/G_a$ ; We note that  $F_1 < F_2 < F_3 < F_4$ ;

From the 4 combinations that meet the gain requirement, we must compare only (1,4) and (2,3) because  $F_2 + (F_3 - 1)/G_2 < F_2 + (F_4 - 1)/G_2$  and  $F_2 + (F_3 - 1)/G_2 < F_3 + (F_4 - 1)/G_3$

$F_1 = 0.56dB = 1.138$ ,  $F_2 = 0.81dB = 1.205$ ,  $F_3 = 0.93dB = 1.239$ ,  $F_4 = 1.22dB = 1.324$ ,  $G_1 = 6.8dB = 4.786$ ,  $G_2 = 7.6dB = 5.754$ ;  $F(1,4) = 1.138 + (1.324 - 1)/4.786 = 1.205 = 0.81dB$ ;  $F(2,3) = 1.205 + (1.239 - 1)/5.754 = 1.261 = 1.01dB$ ;

$F(1,4) < F(2,3) \rightarrow$  For minimal noise factor we must use devices 1,4

6. a) The transistor can be conjugately matched for maximum gain only if it is unconditionally stable:

$|S_{11}| = 0.641 < 1$  ;  $|S_{22}| = 0.520 < 1$  ;  $K = 1.102 > 1$  ;  $|\Delta| = |(-0.122) + j \cdot (0.252)| = 0.280 < 1$

b\_1)  $G_{Tmax} = |S_{21}|/|S_{12}| \cdot [K - \sqrt{(K^2 - 1)}] = 21.61 = 13.35dB$

b\_2) Complex calculus from C8/2017, S106:

$B_1 = 1.062$  ;  $C_1 = (-0.523) + j \cdot (-0.032)$  ;  $\Gamma_S = (-0.852) + j \cdot (0.052) = 0.853 \angle 176.5^\circ$

$B_2 = 0.781$  ;  $C_2 = (-0.212) + j \cdot (-0.317)$  ;  $\Gamma_L = (-0.447) + j \cdot (0.669) = 0.805 \angle 123.7^\circ$

c) Complex calculus from C7/2017, S28÷34, 2 solutions for the input/output match,  $Z_0 = 50\Omega$  lines

input:  $\theta_{S1} = 166.0^\circ$  ;  $\text{Im}(y_S) = -3.276$  ;  $\theta_{p1} = 107.0^\circ$  or  $\theta_{S2} = 17.4^\circ$  ;  $\text{Im}(y_S) = 3.276$  ;  $\theta_{p2} = 73.0^\circ$

output:  $\theta_{L1} = 9.9^\circ$  ;  $\text{Im}(y_L) = -2.713$  ;  $\theta_{p1} = 110.2^\circ$  or  $\theta_{L2} = 46.3^\circ$  ;  $\text{Im}(y_L) = 2.713$  ;  $\theta_{p2} = 69.8^\circ$

d) Second stage is identical to the first one, we can reuse c) results. There are 4 solutions (any offers the points) from the first stage output towards the second stage input (Pr.2017, C10/2017, S76÷84):

d1)  $\theta_{L1} = 9.9^\circ$  ;  $\text{Im}(y_L) = -2.713 + (-3.276) = -5.989$ ;  $\theta_{p1} = 99.5^\circ$  ;  $\theta_{S1} = 166.0^\circ$  ;

d2)  $\theta_{L2} = 46.3^\circ$  ;  $\text{Im}(y_L) = 2.713 + (-3.276) = -0.562$ ;  $\theta_{p2} = 150.6^\circ$  ;  $\theta_{S1} = 166.0^\circ$  ;

d3)  $\theta_{L1} = 9.9^\circ$  ;  $\text{Im}(y_L) = -2.713 + (3.276) = 0.562$ ;  $\theta_{p3} = 29.4^\circ$  ;  $\theta_{S2} = 17.4^\circ$  ;

d4)  $\theta_{L2} = 46.3^\circ$  ;  $\text{Im}(y_L) = 2.713 + (3.276) = 5.989$ ;  $\theta_{p4} = 80.5^\circ$  ;  $\theta_{S2} = 17.4^\circ$  ;

e) We note that the electrical length  $\theta = \beta \cdot l$  is proportional to the physical length and the layout is T shaped (series lines are perpendicular to the shunt stub),  $\Sigma \theta_{serie} \times \theta_{paralel} \sim$  Substrate Area. We must compute all solutions for d) and compare individual products  $\Sigma \theta_{serie} \times \theta_{paralel}$ .

e1)  $\theta_s = 9.9 + 166.0 = 176.0$  ;  $\theta_p = 99.5$  ;  $A \sim 17503.9$

e2)  $\theta_s = 46.3 + 166.0 = 212.3$  ;  $\theta_p = 150.6$  ;  $A \sim 31990.4$

e3)  $\theta_s = 9.9 + 17.4 = 27.4$  ;  $\theta_p = 29.4$  ;  $A \sim 803.1$

e4)  $\theta_s = 46.3 + 17.4 = 63.8$  ;  $\theta_p = 80.5$  ;  $A \sim 5133.8$

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