

### Subject no. 1

1.  $Z_L = 37\Omega$  parallel with  $1.15\text{nH}$  inductor at  $7.1\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0270\text{S} + j \cdot (-0.0195)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 1.351 + j \cdot (-0.975)$  then b)  $z = 1/y = 0.487 + j \cdot (0.351)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.880 + j \cdot 1.020 - 1) / (0.880 + j \cdot 1.020 + 1)$

$\Gamma = (0.178) + j \cdot (0.446) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.480 \angle 68.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $26\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.3\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 36.056 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.9\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4f_1) = \pi/2 \cdot f_2/f_1 = 0.199 \cdot \pi = 0.624$ ;  $\tan(\beta \cdot l) = 0.720$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 31.10\Omega + j \cdot (9.81)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 22.571\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/7) = 2\pi/7$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 1.065\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -154.1^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 143.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 141.9^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 12.2^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 170.2^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 46.8^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.059 = 4.856\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.856\text{dB} + 1.565\text{dB} = 8.709\text{dB}$

d)  $P_{in} = 135\mu\text{W} = -8.697\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -8.697\text{dBm} + 8.709\text{dB} = 0.012\text{dBm} = 1.003\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.177 > 1$ ;  $|\Delta| = |(0.229) + j \cdot (0.174)| = 0.288 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### Subject no. 2

1.  $Z_L = 50\Omega$  series with  $0.78\text{nH}$  inductor at  $8.5\text{GHz}$ . It's easier to compute first: b)  $Z_L = 50.00\Omega + j \cdot (41.66)\Omega$ ,  $z = Z_L/50\Omega = 1.000 + j \cdot (0.833)$  then a)  $y = 1/z = 0.590 + j \cdot (-0.492)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.730 - j \cdot 0.990 - 1) / (0.730 - j \cdot 0.990 + 1)$

$\Gamma = (0.129) + j \cdot (-0.498) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.515 \angle -75.5^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $50\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 50.000 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 3.6\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4f_1) = \pi/2 \cdot f_2/f_1 = 0.200 \cdot \pi = 0.628$ ;  $\tan(\beta \cdot l) = 0.727$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 50.00\Omega + j \cdot (0.00)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 20.156\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/10) = 2\pi/10$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.463\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.692 \angle 167.8^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 118.3^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 163.0^\circ$ ;  $\text{Im}(y_S) = -1.917$ ;  $\theta_{p1} = 117.5^\circ$  or  $\theta_{S2} = 29.2^\circ$ ;  $\text{Im}(y_S) = 1.917$ ;  $\theta_{p2} = 62.5^\circ$

output:  $\theta_{L1} = 2.9^\circ$ ;  $\text{Im}(y_L) = -1.352$ ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 58.8^\circ$ ;  $\text{Im}(y_L) = 1.352$ ;  $\theta_{p2} = 53.5^\circ$

- b) The shunt stubs **must** be placed in parallel with the 50Ω source/load  
c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.919 = 2.830\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.865 = 5.872\text{dB}$ ;  
The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.830\text{dB} + 5.872\text{dB} + 1.634\text{dB} = 10.336\text{dB}$   
d)  $P_{\text{in}} = 135\mu\text{W} = -8.697\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -8.697\text{dBm} + 10.336\text{dB} = 1.640\text{dBm} = 1.459\text{mW}$   
e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.692 < 1$ ;  $|S_{22}| = 0.560 < 1$ ;  $K = 1.062 > 1$ ;  $|\Delta| = |(-0.049) + j \cdot (0.352)| = 0.356 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### Subject no. 3

1.  $Z_L = 25\Omega$  series with 1.00nH inductor at 9.4GHz . It's easier to compute first: b)  $Z_L = 25.00\Omega + j \cdot (59.06)\Omega$ ,  $z = Z_L/50\Omega = 0.500 + j \cdot (1.181)$  then a)  $y = 1/z = 0.304 + j \cdot (-0.718)$   
2. a)  $\Gamma = (z - 1) / (z + 1) = (0.895 - j \cdot 0.750 - 1) / (0.895 - j \cdot 0.750 + 1)$   
 $\Gamma = (0.088) + j \cdot (-0.361) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.372 \angle -76.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$   
3. The quarter wave transformer is designed to match a 41Ω load to a 50Ω source at  $f_1 = 6.7\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 45.277 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .  
At  $f_2 = 3.9\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.291 \cdot \pi = 0.914$ ;  $\tan(\beta \cdot l) = 1.298$   
 $Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 46.22\Omega + j \cdot (4.44)\Omega$   
4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 35.806\Omega$ .  
For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/9) = 2\pi/9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.266\text{pF}$   
5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.688 \angle -103.7^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.298 \angle -153.2^\circ$   
Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$   
input:  $\theta_{S1} = 118.6^\circ$ ;  $\text{Im}(y_S) = -1.896$ ;  $\theta_{p1} = 117.8^\circ$  or  $\theta_{S2} = 165.1^\circ$ ;  $\text{Im}(y_S) = 1.896$ ;  $\theta_{p2} = 62.2^\circ$   
output:  $\theta_{L1} = 130.3^\circ$ ;  $\text{Im}(y_L) = -0.624$ ;  $\theta_{p1} = 148.0^\circ$  or  $\theta_{L2} = 22.9^\circ$ ;  $\text{Im}(y_L) = 0.624$ ;  $\theta_{p2} = 32.0^\circ$   
b) The shunt stubs **must** be placed in parallel with the 50Ω source/load  
c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.899 = 2.785\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.097 = 0.404\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.843 = 4.537\text{dB}$ ;  
The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.785\text{dB} + 4.537\text{dB} + 0.404\text{dB} = 7.726\text{dB}$   
d)  $P_{\text{in}} = 105\mu\text{W} = -9.788\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -9.788\text{dBm} + 7.726\text{dB} = -2.062\text{dBm} = 0.622\text{mW}$   
e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.688 < 1$ ;  $|S_{22}| = 0.298 < 1$ ;  $K = 1.118 > 1$ ;  $|\Delta| = |(-0.259) + j \cdot (-0.109)| = 0.281 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### Subject no. 4

1.  $Z_L = 63\Omega$  paralel with 1.13nH inductor at 9.5GHz . It's easier to compute first: a)  $Y_L = 0.0159\text{S} + j \cdot (-0.0148)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 0.794 + j \cdot (-0.741)$  then b)  $z = 1/y = 0.673 + j \cdot (0.629)$   
2. a)  $\Gamma = (z - 1) / (z + 1) = (1.025 - j \cdot 1.000 - 1) / (1.025 - j \cdot 1.000 + 1)$   
 $\Gamma = (0.206) + j \cdot (-0.392) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.443 \angle -62.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$   
3. The quarter wave transformer is designed to match a 37Ω load to a 50Ω source at  $f_1 = 7.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 43.012 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .  
At  $f_2 = 4.0\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.286 \cdot \pi = 0.898$ ;  $\tan(\beta \cdot l) = 1.254$   
 $Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 43.99\Omega + j \cdot (6.48)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 26.049\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 7) = 2\pi / 7$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.594\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.660 \angle -173.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.555 \angle 135.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 152.1^\circ$ ;  $\text{Im}(y_S) = -1.757$ ;  $\theta_{p1} = 119.6^\circ$  or  $\theta_{S2} = 20.9^\circ$ ;  $\text{Im}(y_S) = 1.757$ ;  $\theta_{p2} = 60.4^\circ$

output:  $\theta_{L1} = 174.4^\circ$ ;  $\text{Im}(y_L) = -1.334$ ;  $\theta_{p1} = 126.8^\circ$  or  $\theta_{L2} = 50.6^\circ$ ;  $\text{Im}(y_L) = 1.334$ ;  $\theta_{p2} = 53.2^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.772 = 2.484\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.445 = 1.599\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.276 = 5.154\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.484\text{dB} + 5.154\text{dB} + 1.599\text{dB} = 9.237\text{dB}$

d)  $P_{in} = 130\mu\text{W} = -8.861\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -8.861\text{dBm} + 9.237\text{dB} = 0.376\text{dBm} = 1.090\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.660 < 1$ ;  $|S_{22}| = 0.555 < 1$ ;  $K = 1.125 > 1$ ;  $|\Delta| = |(0.144) + j \cdot (0.299)| = 0.332 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 5**

1.  $Z_L = 61\Omega$  series with  $1.32\text{nH}$  inductor at  $6.5\text{GHz}$ . It's easier to compute first: b)  $Z_L = 61.00\Omega + j \cdot (53.91)\Omega$ ,  $z = Z_L / 50\Omega = 1.220 + j \cdot (1.078)$  then a)  $y = 1/z = 0.460 + j \cdot (-0.407)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.290 + j \cdot 0.970 - 1) / (1.290 + j \cdot 0.970 + 1)$

$\Gamma = (0.259) + j \cdot (0.314) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.407 \angle 50.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $35\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.1\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 41.833\Omega$  and the line physical length is  $l_1 = \lambda_1 / 4 = c / 4 / f_1$ .

At  $f_2 = 2.2\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c / f_2) \cdot (c / 4 / f_1) = \pi / 2 \cdot f_2 / f_1 = 0.155 \cdot \pi = 0.487$ ;  $\tan(\beta \cdot l) = 0.529$

$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 37.46\Omega + j \cdot (5.55)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 24.027\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 13) = 2\pi / 13$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.241\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle 172.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 128.2^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 177.9^\circ$ ;  $\text{Im}(y_L) = -1.352$ ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 53.9^\circ$ ;  $\text{Im}(y_L) = 1.352$ ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.501 = 5.441\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.441\text{dB} + 1.634\text{dB} = 9.771\text{dB}$

d)  $P_{in} = 140\mu\text{W} = -8.539\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -8.539\text{dBm} + 9.771\text{dB} = 1.232\text{dBm} = 1.328\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$ ;  $|S_{22}| = 0.560 < 1$ ;  $K = 1.093 > 1$ ;  $|\Delta| = |(0.031) + j \cdot (0.367)| = 0.369 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 6**

1.  $Z_L = 25\Omega$  paralel with 0.56pF capacitor at 7.2GHz . It's easier to compute first: a)  $Y_L = 0.0400S + j \cdot (0.0253)S$ ,  $y = Y_L \cdot 50\Omega = 2.000 + j \cdot (1.267)$  then b)  $z = 1/y = 0.357 + j \cdot (-0.226)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.740 - j \cdot 1.055 - 1) / (0.740 - j \cdot 1.055 + 1)$

$\Gamma = (0.160) + j \cdot (-0.510) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.534 \angle -72.6^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $50\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.7\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 50.000 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.4\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.156 \cdot \pi = 0.490$ ;  $\tan(\beta \cdot l) = 0.533$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 50.00\Omega + j \cdot (0.00)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 25.908\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/6) = 2\pi/6$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 1.400\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle 172.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 130.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 177.0^\circ$ ;  $\text{Im}(y_L) = -1.352$ ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 53.0^\circ$ ;  $\text{Im}(y_L) = 1.352$ ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.437 = 5.362\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.362\text{dB} + 1.634\text{dB} = 9.692\text{dB}$

d)  $P_{in} = 75\mu\text{W} = -11.249\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -11.249\text{dBm} + 9.692\text{dB} = -1.558\text{dBm} = 0.699\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$ ;  $|S_{22}| = 0.560 < 1$ ;  $K = 1.091 > 1$ ;  $|\Delta| = |(0.042) + j \cdot (0.372)| = 0.374 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 7**

1.  $Z_L = 58\Omega$  paralel with 0.26pF capacitor at 9.6GHz . It's easier to compute first: a)  $Y_L = 0.0172S + j \cdot (0.0157)S$ ,  $y = Y_L \cdot 50\Omega = 0.862 + j \cdot (0.784)$  then b)  $z = 1/y = 0.635 + j \cdot (-0.577)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.725 - j \cdot 0.800 - 1) / (0.725 - j \cdot 0.800 + 1)$

$\Gamma = (0.046) + j \cdot (-0.443) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.445 \angle -84.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $43\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.5\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 46.368 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.0\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.105 \cdot \pi = 0.331$ ;  $\tan(\beta \cdot l) = 0.343$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 43.64\Omega + j \cdot (2.02)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 29.345\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/14) = 2\pi/14$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.179\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -158.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 140.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 143.9^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 14.1^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 171.7^\circ$  ;  $\text{Im}(y_L) = -1.317$  ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 48.3^\circ$  ;  $\text{Im}(y_L) = 1.317$  ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.119 = 4.940\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.940\text{dB} + 1.565\text{dB} = 8.793\text{dB}$

d)  $P_{\text{in}} = 110\mu\text{W} = -9.586\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -9.586\text{dBm} + 8.793\text{dB} = -0.793\text{dBm} = 0.833\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.173 > 1$  ;  $|\Delta| = |(0.208) + j \cdot (0.204)| = 0.292 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 8**

1.  $Z_L = 57\Omega$  parallel with  $0.55\text{nH}$  inductor at  $8.6\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0175\text{S} + j \cdot (-0.0336)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 0.877 + j \cdot (-1.682)$  then b)  $z = 1/y = 0.244 + j \cdot (0.467)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.190 + j \cdot 1.025 - 1) / (1.190 + j \cdot 1.025 + 1)$

$\Gamma = (0.251) + j \cdot (0.351) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.431 \angle 54.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $72\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.8\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 60.000\Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 3.8\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.216 \cdot \pi = 0.678$ ;  $\tan(\beta \cdot l) = 0.806$

$Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 61.37\Omega + j \cdot (-10.99)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 32.787\Omega$ .

For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$  ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/14) = 2\pi/14$  ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.231\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$  ;  $\Gamma_S = 0.680 \angle 172.0^\circ$  ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$  ;  $\Gamma_L = 0.560 \angle 127.3^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$  ;  $\text{Im}(y_S) = -1.855$  ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$  ;  $\text{Im}(y_S) = 1.855$  ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 178.4^\circ$  ;  $\text{Im}(y_L) = -1.352$  ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 54.3^\circ$  ;  $\text{Im}(y_L) = 1.352$  ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.534 = 5.483\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.483\text{dB} + 1.634\text{dB} = 9.813\text{dB}$

d)  $P_{\text{in}} = 120\mu\text{W} = -9.208\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -9.208\text{dBm} + 9.813\text{dB} = 0.605\text{dBm} = 1.149\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$  ;  $|S_{22}| = 0.560 < 1$  ;  $K = 1.094 > 1$  ;  $|\Delta| = |(0.026) + j \cdot (0.365)| = 0.366 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 9**

1.  $Z_L = 68\Omega$  series with  $0.49\text{nH}$  inductor at  $8.3\text{GHz}$ . It's easier to compute first: b)  $Z_L = 68.00\Omega + j \cdot (25.55)\Omega$ ,  $z = Z_L/50\Omega = 1.360 + j \cdot (0.511)$  then a)  $y = 1/z = 0.644 + j \cdot (-0.242)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.815 - j \cdot 1.280 - 1) / (0.815 - j \cdot 1.280 + 1)$

$\Gamma = (0.264) + j \cdot (-0.519) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.582 \angle -63.0^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $63\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.1\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 56.125\Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 2.7\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.190 \cdot \pi = 0.597$ ;  $\tan(\beta \cdot l) = 0.680$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 58.21\Omega + j \cdot (-6.27)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 24.450\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 9) = 2\pi / 9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.587\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -148.9^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 147.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 139.3^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 9.6^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 168.2^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 44.8^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.979 = 4.741\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.741\text{dB} + 1.565\text{dB} = 8.594\text{dB}$

d)  $P_{in} = 125\mu\text{W} = -9.031\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -9.031\text{dBm} + 8.594\text{dB} = -0.437\text{dBm} = 0.904\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.184 > 1$ ;  $|\Delta| = |(0.251) + j \cdot (0.130)| = 0.283 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 10**

1.  $Z_L = 55\Omega$  parallel with  $0.71\text{nH}$  inductor at  $8.5\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0182\text{S} + j \cdot (-0.0264)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 0.909 + j \cdot (-1.319)$  then b)  $z = 1/y = 0.354 + j \cdot (0.514)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.935 - j \cdot 0.740 - 1) / (0.935 - j \cdot 0.740 + 1)$

$\Gamma = (0.098) + j \cdot (-0.345) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.359 \angle -74.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $48\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.5\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 48.990\Omega$  and the line physical length is  $l_1 = \lambda_1 / 4 = c / 4 / f_1$ .

At  $f_2 = 3.5\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c / f_2) \cdot (c / 4 / f_1) = \pi / 2 \cdot f_2 / f_1 = 0.184 \cdot \pi = 0.579$ ;  $\tan(\beta \cdot l) = 0.653$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 48.58\Omega + j \cdot (0.91)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 38.534\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 9) = 2\pi / 9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.271\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle 172.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 124.6^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 179.7^\circ$ ;  $\text{Im}(y_L) = -1.352$ ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 55.7^\circ$ ;  $\text{Im}(y_L) = 1.352$ ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.633 = 5.602\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.602\text{dB} + 1.634\text{dB} = 9.932\text{dB}$

d)  $P_{in} = 120\mu\text{W} = -9.208\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -9.208\text{dBm} + 9.932\text{dB} = 0.724\text{dBm} = 1.181\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$ ;  $|S_{22}| = 0.560 < 1$ ;  $K = 1.100 > 1$ ;  $|\Delta| = |(0.011) + j \cdot (0.358)| = 0.358 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 11**

1.  $Z_L = 50\Omega$  series with  $0.99\text{nH}$  inductor at  $7.0\text{GHz}$ . It's easier to compute first: b)  $Z_L = 50.00\Omega + j \cdot (43.54)\Omega$ ,  $z = Z_L/50\Omega = 1.000 + j \cdot (0.871)$  then a)  $y = 1/z = 0.569 + j \cdot (-0.495)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.925 + j \cdot 1.175 - 1) / (0.925 + j \cdot 1.175 + 1)$

$\Gamma = (0.243) + j \cdot (0.462) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.522 \angle 62.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $27\Omega$  load to a  $50\Omega$  source at  $f_1 = 10.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 36.742 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 4.1\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.205 \cdot \pi = 0.644$ ;  $\tan(\beta \cdot l) = 0.751$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 32.37\Omega + j \cdot (9.73)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 33.622\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/7) = 2\pi/7$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.530\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.676 \angle -106.4^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.286 \angle -157.4^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 119.5^\circ$ ;  $\text{Im}(y_S) = -1.835$ ;  $\theta_{p1} = 118.6^\circ$  or  $\theta_{S2} = 166.9^\circ$ ;  $\text{Im}(y_S) = 1.835$ ;  $\theta_{p2} = 61.4^\circ$

output:  $\theta_{L1} = 132.0^\circ$ ;  $\text{Im}(y_L) = -0.597$ ;  $\theta_{p1} = 149.2^\circ$  or  $\theta_{L2} = 25.4^\circ$ ;  $\text{Im}(y_L) = 0.597$ ;  $\theta_{p2} = 30.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.842 = 2.652\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.089 = 0.371\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.965 = 4.721\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.652\text{dB} + 4.721\text{dB} + 0.371\text{dB} = 7.743\text{dB}$

d)  $P_{in} = 55\mu\text{W} = -12.596\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -12.596\text{dBm} + 7.743\text{dB} = -4.853\text{dBm} = 0.327\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.676 < 1$ ;  $|S_{22}| = 0.286 < 1$ ;  $K = 1.155 > 1$ ;  $|\Delta| = |(-0.240) + j \cdot (-0.119)| = 0.268 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 12**

1.  $Z_L = 32\Omega$  series with  $0.38\text{pF}$  capacitor at  $8.9\text{GHz}$ . It's easier to compute first: b)  $Z_L = 32.00\Omega + j \cdot (-47.06)\Omega$ ,  $z = Z_L/50\Omega = 0.640 + j \cdot (-0.941)$  then a)  $y = 1/z = 0.494 + j \cdot (0.727)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.025 - j \cdot 0.925 - 1) / (1.025 - j \cdot 0.925 + 1)$

$\Gamma = (0.183) + j \cdot (-0.373) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.416 \angle -63.9^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $57\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.8\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 53.385 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 3.4\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.193 \cdot \pi = 0.607$ ;  $\tan(\beta \cdot l) = 0.694$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 54.52\Omega + j \cdot (-3.35)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 18.550\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/13) = 2\pi/13$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.617\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle 172.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 126.4^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 178.8^\circ$  ;  $\text{Im}(y_L) = -1.352$  ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 54.8^\circ$  ;  $\text{Im}(y_L) = 1.352$  ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.565 = 5.520\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.520\text{dB} + 1.634\text{dB} = 9.850\text{dB}$

d)  $P_{\text{in}} = 50\mu\text{W} = -13.010\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -13.010\text{dBm} + 9.850\text{dB} = -3.161\text{dBm} = 0.483\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$  ;  $|S_{22}| = 0.560 < 1$  ;  $K = 1.096 > 1$  ;  $|\Delta| = |(0.021) + j \cdot (0.363)| = 0.363 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 13**

1.  $Z_L = 67\Omega$  series with  $0.25\text{pF}$  capacitor at  $8.9\text{GHz}$ . It's easier to compute first: b)  $Z_L = 67.00\Omega + j \cdot (-71.53)\Omega$ ,  $z = Z_L/50\Omega = 1.340 + j \cdot (-1.431)$  then a)  $y = 1/z = 0.349 + j \cdot (0.372)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.720 + j \cdot 1.185 - 1) / (0.720 + j \cdot 1.185 + 1)$

$\Gamma = (0.211) + j \cdot (0.543) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.583 \angle 68.7^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $47\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 48.477 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 2.9\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.161 \cdot \pi = 0.506$ ;  $\tan(\beta \cdot l) = 0.554$

$Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 47.67\Omega + j \cdot (1.25)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 36.018\Omega$ .

For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$  ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/8) = 2\pi/8$  ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.388\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$  ;  $\Gamma_S = 0.684 \angle 170.6^\circ$  ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$  ;  $\Gamma_L = 0.560 \angle 120.1^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 161.3^\circ$  ;  $\text{Im}(y_S) = -1.875$  ;  $\theta_{p1} = 118.1^\circ$  or  $\theta_{S2} = 28.1^\circ$  ;  $\text{Im}(y_S) = 1.875$  ;  $\theta_{p2} = 61.9^\circ$

output:  $\theta_{L1} = 2.0^\circ$  ;  $\text{Im}(y_L) = -1.352$  ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 57.9^\circ$  ;  $\text{Im}(y_L) = 1.352$  ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.879 = 2.740\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.795 = 5.792\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.740\text{dB} + 5.792\text{dB} + 1.634\text{dB} = 10.166\text{dB}$

d)  $P_{\text{in}} = 70\mu\text{W} = -11.549\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -11.549\text{dBm} + 10.166\text{dB} = -1.383\text{dBm} = 0.727\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.684 < 1$  ;  $|S_{22}| = 0.560 < 1$  ;  $K = 1.095 > 1$  ;  $|\Delta| = |(-0.020) + j \cdot (0.350)| = 0.350 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 14**

1.  $Z_L = 57\Omega$  series with  $0.54\text{nH}$  inductor at  $9.7\text{GHz}$ . It's easier to compute first: b)  $Z_L = 57.00\Omega + j \cdot (32.91)\Omega$ ,  $z = Z_L/50\Omega = 1.140 + j \cdot (0.658)$  then a)  $y = 1/z = 0.658 + j \cdot (-0.380)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.050 + j \cdot 1.100 - 1) / (1.050 + j \cdot 1.100 + 1)$

$\Gamma = (0.242) + j \cdot (0.406) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.473 \angle 59.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $62\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 55.678 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 3.7\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.264 \cdot \pi = 0.830$ ;  $\tan(\beta \cdot l) = 1.094$



$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 54.83\Omega + j \cdot (-5.88)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 18.363\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega \cdot C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 14) = 2\pi / 14$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.373\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -145.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 150.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 137.4^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 7.6^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 166.7^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 43.3^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.921 = 4.655\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.655\text{dB} + 1.565\text{dB} = 8.508\text{dB}$

d)  $P_{in} = 125\mu\text{W} = -9.031\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -9.031\text{dBm} + 8.508\text{dB} = -0.523\text{dBm} = 0.887\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.189 > 1$ ;  $|\Delta| = |(0.262) + j \cdot (0.095)| = 0.279 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 15**

1.  $Z_L = 56\Omega$  series with  $0.37\text{pF}$  capacitor at  $7.0\text{GHz}$ . It's easier to compute first: b)  $Z_L = 56.00\Omega + j \cdot (-61.45)\Omega$ ,  $z = Z_L / 50\Omega = 1.120 + j \cdot (-1.229)$  then a)  $y = 1/z = 0.405 + j \cdot (0.445)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.755 + j \cdot 1.020 - 1) / (0.755 + j \cdot 1.020 + 1)$

$\Gamma = (0.148) + j \cdot (0.495) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.517 \angle 73.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $53\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.9\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 51.478\Omega$  and the line physical length is  $l_1 = \lambda_1 / 4 = c / 4 / f_1$ .

At  $f_2 = 4.0\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c / f_2) \cdot (c / 4 / f_1) = \pi / 2 \cdot f_2 / f_1 = 0.202 \cdot \pi = 0.635$ ;  $\tan(\beta \cdot l) = 0.736$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 51.91\Omega + j \cdot (-1.44)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 27.832\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega \cdot C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 10) = 2\pi / 10$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.301\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.688 \angle 169.2^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 119.2^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 162.1^\circ$ ;  $\text{Im}(y_S) = -1.896$ ;  $\theta_{p1} = 117.8^\circ$  or  $\theta_{S2} = 28.7^\circ$ ;  $\text{Im}(y_S) = 1.896$ ;  $\theta_{p2} = 62.2^\circ$

output:  $\theta_{L1} = 2.4^\circ$ ;  $\text{Im}(y_L) = -1.352$ ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 58.4^\circ$ ;  $\text{Im}(y_L) = 1.352$ ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.899 = 2.785\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.830 = 5.832\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.785\text{dB} + 5.832\text{dB} + 1.634\text{dB} = 10.251\text{dB}$

d)  $P_{in} = 55\mu\text{W} = -12.596\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -12.596\text{dBm} + 10.251\text{dB} = -2.346\text{dBm} = 0.583\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.688 < 1$ ;  $|S_{22}| = 0.560 < 1$ ;  $K = 1.079 > 1$ ;  $|\Delta| = |(-0.034) + j \cdot (0.351)| = 0.353 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 16**

1.  $Z_L = 48\Omega$  paralel with 0.30pF capacitor at 7.5GHz . It's easier to compute first: a)  $Y_L = 0.0208S + j \cdot (0.0141)S$ ,  $y = Y_L \cdot 50\Omega = 1.042 + j \cdot (0.707)$  then b)  $z = 1/y = 0.657 + j \cdot (-0.446)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.265 - j \cdot 1.250 - 1) / (1.265 - j \cdot 1.250 + 1)$

$\Gamma = (0.323) + j \cdot (-0.374) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.494 \angle -49.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $33\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.1\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 40.620 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 4.3\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.265 \cdot \pi = 0.834$ ;  $\tan(\beta \cdot l) = 1.102$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 40.56\Omega + j \cdot (8.45)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 19.540\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/11) = 2\pi/11$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.400\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.684 \angle -104.6^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.294 \angle -154.6^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 118.9^\circ$ ;  $\text{Im}(y_S) = -1.875$ ;  $\theta_{p1} = 118.1^\circ$  or  $\theta_{S2} = 165.7^\circ$ ;  $\text{Im}(y_S) = 1.875$ ;  $\theta_{p2} = 61.9^\circ$

output:  $\theta_{L1} = 130.8^\circ$ ;  $\text{Im}(y_L) = -0.615$ ;  $\theta_{p1} = 148.4^\circ$  or  $\theta_{L2} = 23.8^\circ$ ;  $\text{Im}(y_L) = 0.615$ ;  $\theta_{p2} = 31.6^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.879 = 2.740\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.095 = 0.393\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.883 = 4.599\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.740\text{dB} + 4.599\text{dB} + 0.393\text{dB} = 7.731\text{dB}$

d)  $P_{in} = 145\mu\text{W} = -8.386\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -8.386\text{dBm} + 7.731\text{dB} = -0.655\text{dBm} = 0.860\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.684 < 1$ ;  $|S_{22}| = 0.294 < 1$ ;  $K = 1.130 > 1$ ;  $|\Delta| = |(-0.252) + j \cdot (-0.113)| = 0.276 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 17**

1.  $Z_L = 59\Omega$  series with 1.16nH inductor at 9.5GHz . It's easier to compute first: b)  $Z_L = 59.00\Omega + j \cdot (69.24)\Omega$ ,  $z = Z_L/50\Omega = 1.180 + j \cdot (1.385)$  then a)  $y = 1/z = 0.356 + j \cdot (-0.418)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.070 - j \cdot 0.865 - 1) / (1.070 - j \cdot 0.865 + 1)$

$\Gamma = (0.177) + j \cdot (-0.344) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.387 \angle -62.7^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $71\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.3\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 59.582 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.8\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.192 \cdot \pi = 0.602$ ;  $\tan(\beta \cdot l) = 0.688$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 62.56\Omega + j \cdot (-10.30)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 33.806\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/14) = 2\pi/14$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.160\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle 172.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 125.5^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 179.3^\circ$  ;  $\text{Im}(y_L) = -1.352$  ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 55.2^\circ$  ;  $\text{Im}(y_L) = 1.352$  ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.599 = 5.561\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.561\text{dB} + 1.634\text{dB} = 9.891\text{dB}$

d)  $P_{\text{in}} = 80\mu\text{W} = -10.969\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -10.969\text{dBm} + 9.891\text{dB} = -1.078\text{dBm} = 0.780\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$  ;  $|S_{22}| = 0.560 < 1$  ;  $K = 1.098 > 1$  ;  $|\Delta| = |(0.016) + j \cdot (0.360)| = 0.361 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 18**

1.  $Z_L = 28\Omega$  series with  $1.14\text{nH}$  inductor at  $7.5\text{GHz}$ . It's easier to compute first: b)  $Z_L = 28.00\Omega + j \cdot (53.72)\Omega$ ,  $z = Z_L/50\Omega = 0.560 + j \cdot (1.074)$  then a)  $y = 1/z = 0.381 + j \cdot (-0.732)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.865 + j \cdot 1.135 - 1) / (0.865 + j \cdot 1.135 + 1)$

$\Gamma = (0.217) + j \cdot (0.476) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.524 \angle 65.5^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $27\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.4\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 36.742 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 2.8\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.189 \cdot \pi = 0.594$ ;  $\tan(\beta \cdot l) = 0.676$

$Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 31.55\Omega + j \cdot (9.16)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 22.704\Omega$ .

For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$  ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/8) = 2\pi/8$  ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.701\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$  ;  $\Gamma_S = 0.692 \angle -102.8^\circ$  ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$  ;  $\Gamma_L = 0.302 \angle -151.8^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 118.3^\circ$  ;  $\text{Im}(y_S) = -1.917$  ;  $\theta_{p1} = 117.5^\circ$  or  $\theta_{S2} = 164.5^\circ$  ;  $\text{Im}(y_S) = 1.917$  ;  $\theta_{p2} = 62.5^\circ$

output:  $\theta_{L1} = 129.7^\circ$  ;  $\text{Im}(y_L) = -0.634$  ;  $\theta_{p1} = 147.6^\circ$  or  $\theta_{L2} = 22.1^\circ$  ;  $\text{Im}(y_L) = 0.634$  ;  $\theta_{p2} = 32.4^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.919 = 2.830\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.100 = 0.415\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.802 = 4.475\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.830\text{dB} + 4.475\text{dB} + 0.415\text{dB} = 7.721\text{dB}$

d)  $P_{\text{in}} = 120\mu\text{W} = -9.208\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -9.208\text{dBm} + 7.721\text{dB} = -1.487\text{dBm} = 0.710\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.692 < 1$  ;  $|S_{22}| = 0.302 < 1$  ;  $K = 1.106 > 1$  ;  $|\Delta| = |(-0.265) + j \cdot (-0.105)| = 0.285 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 19**

1.  $Z_L = 35\Omega$  parallel with  $0.60\text{nH}$  inductor at  $8.2\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0286\text{S} + j \cdot (-0.0323)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 1.429 + j \cdot (-1.617)$  then b)  $z = 1/y = 0.307 + j \cdot (0.347)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.235 - j \cdot 0.760 - 1) / (1.235 - j \cdot 0.760 + 1)$

$\Gamma = (0.198) + j \cdot (-0.273) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.337 \angle -54.0^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $26\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.1\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 36.056 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 2.9\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.179 \cdot \pi = 0.562$ ;  $\tan(\beta \cdot l) = 0.630$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 30.11\Omega + j \cdot (9.04)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 31.623\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 9) = 2\pi / 9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.285\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -146.3^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 149.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 138.0^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 8.3^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 167.2^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 43.8^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.941 = 4.685\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.685\text{dB} + 1.565\text{dB} = 8.538\text{dB}$

d)  $P_{in} = 70\mu\text{W} = -11.549\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -11.549\text{dBm} + 8.538\text{dB} = -3.011\text{dBm} = 0.500\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.187 > 1$ ;  $|\Delta| = |(0.259) + j \cdot (0.107)| = 0.280 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 20**

1.  $Z_L = 68\Omega$  parallel with  $0.51\text{nH}$  inductor at  $8.7\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0147\text{S} + j \cdot (-0.0359)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 0.735 + j \cdot (-1.793)$  then b)  $z = 1/y = 0.196 + j \cdot (0.477)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.025 + j \cdot 1.075 - 1) / (1.025 + j \cdot 1.075 + 1)$

$\Gamma = (0.229) + j \cdot (0.409) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.469 \angle 60.7^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $46\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.7\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 47.958\Omega$  and the line physical length is  $l_1 = \lambda_1 / 4 = c / 4 / f_1$ .

At  $f_2 = 3.7\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c / f_2) \cdot (c / 4 / f_1) = \pi / 2 \cdot f_2 / f_1 = 0.191 \cdot \pi = 0.599$ ;  $\tan(\beta \cdot l) = 0.683$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 47.20\Omega + j \cdot (1.83)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 21.880\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 12) = 2\pi / 12$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.286\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle -105.5^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.290 \angle -156.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 119.2^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 166.3^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 131.4^\circ$ ;  $\text{Im}(y_L) = -0.606$ ;  $\theta_{p1} = 148.8^\circ$  or  $\theta_{L2} = 24.6^\circ$ ;  $\text{Im}(y_L) = 0.606$ ;  $\theta_{p2} = 31.2^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.092 = 0.382\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.924 = 4.660\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 4.660\text{dB} + 0.382\text{dB} = 7.737\text{dB}$

d)  $P_{in} = 75\mu\text{W} = -11.249\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -11.249\text{dBm} + 7.737\text{dB} = -3.513\text{dBm} = 0.445\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$ ;  $|S_{22}| = 0.290 < 1$ ;  $K = 1.143 > 1$ ;  $|\Delta| = |(-0.246) + j \cdot (-0.116)| = 0.272 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 21**

1.  $Z_L = 31\Omega$  series with 1.09nH inductor at 8.4GHz . It's easier to compute first: b)  $Z_L = 31.00\Omega + j \cdot (57.53)\Omega$ ,  $z = Z_L/50\Omega = 0.620 + j \cdot (1.151)$  then a)  $y = 1/z = 0.363 + j \cdot (-0.674)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.005 - j \cdot 0.820 - 1) / (1.005 - j \cdot 0.820 + 1)$

$\Gamma = (0.145) + j \cdot (-0.349) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.379 \angle -67.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $57\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.8\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 53.385 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.5\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.142 \cdot \pi = 0.446$ ;  $\tan(\beta \cdot l) = 0.478$

$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 55.55\Omega + j \cdot (-2.84)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 24.537\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/7) = 2\pi/7$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.594\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.672 \angle 178.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.558 \angle 132.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 157.1^\circ$ ;  $\text{Im}(y_S) = -1.815$ ;  $\theta_{p1} = 118.9^\circ$  or  $\theta_{S2} = 24.9^\circ$ ;  $\text{Im}(y_S) = 1.815$ ;  $\theta_{p2} = 61.1^\circ$

output:  $\theta_{L1} = 176.0^\circ$ ;  $\text{Im}(y_L) = -1.345$ ;  $\theta_{p1} = 126.6^\circ$  or  $\theta_{L2} = 52.0^\circ$ ;  $\text{Im}(y_L) = 1.345$ ;  $\theta_{p2} = 53.4^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.823 = 2.609\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.452 = 1.620\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.371 = 5.277\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.609\text{dB} + 5.277\text{dB} + 1.620\text{dB} = 9.506\text{dB}$

d)  $P_{in} = 100\mu\text{W} = -10.000\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -10.000\text{dBm} + 9.506\text{dB} = -0.494\text{dBm} = 0.893\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.672 < 1$ ;  $|S_{22}| = 0.558 < 1$ ;  $K = 1.103 > 1$ ;  $|\Delta| = |(0.087) + j \cdot (0.346)| = 0.357 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 22**

1.  $Z_L = 56\Omega$  series with 0.54nH inductor at 9.4GHz . It's easier to compute first: b)  $Z_L = 56.00\Omega + j \cdot (31.89)\Omega$ ,  $z = Z_L/50\Omega = 1.120 + j \cdot (0.638)$  then a)  $y = 1/z = 0.674 + j \cdot (-0.384)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.180 + j \cdot 0.920 - 1) / (1.180 + j \cdot 0.920 + 1)$

$\Gamma = (0.221) + j \cdot (0.329) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.396 \angle 56.0^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $34\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.7\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 41.231 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 3.2\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.208 \cdot \pi = 0.653$ ;  $\tan(\beta \cdot l) = 0.765$

$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 38.55\Omega + j \cdot (7.22)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 41.079\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/7) = 2\pi/7$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.540\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.676 \angle 175.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.559 \angle 131.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 158.8^\circ$ ;  $\text{Im}(y_S) = -1.835$ ;  $\theta_{p1} = 118.6^\circ$  or  $\theta_{S2} = 26.2^\circ$ ;  $\text{Im}(y_S) = 1.835$ ;  $\theta_{p2} = 61.4^\circ$

output:  $\theta_{L1} = 176.5^\circ$  ;  $\text{Im}(y_L) = -1.348$  ;  $\theta_{p1} = 126.6^\circ$  or  $\theta_{L2} = 52.5^\circ$  ;  $\text{Im}(y_L) = 1.348$  ;  $\theta_{p2} = 53.4^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.842 = 2.652\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.455 = 1.627\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.404 = 5.320\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.652\text{dB} + 5.320\text{dB} + 1.627\text{dB} = 9.599\text{dB}$

d)  $P_{\text{in}} = 50\mu\text{W} = -13.010\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -13.010\text{dBm} + 9.599\text{dB} = -3.411\text{dBm} = 0.456\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.676 < 1$  ;  $|S_{22}| = 0.559 < 1$  ;  $K = 1.097 > 1$  ;  $|\Delta| = |(0.065) + j \cdot (0.360)| = 0.366 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 23**

1.  $Z_L = 39\Omega$  parallel with  $0.66\text{nH}$  inductor at  $8.9\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0256\text{S} + j \cdot (-0.0271)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 1.282 + j \cdot (-1.355)$  then b)  $z = 1/y = 0.369 + j \cdot (0.389)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.825 + j \cdot 1.075 - 1) / (0.825 + j \cdot 1.075 + 1)$

$\Gamma = (0.186) + j \cdot (0.479) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.514 \angle 68.7^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $68\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.7\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 58.310\Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 3.1\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.160 \cdot \pi = 0.502$ ;  $\tan(\beta \cdot l) = 0.549$

$Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 62.77\Omega + j \cdot (-8.17)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 28.452\Omega$ .

For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$  ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/14) = 2\pi/14$  ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.228\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$  ;  $\Gamma_S = 0.680 \angle 172.0^\circ$  ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$  ;  $\Gamma_L = 0.560 \angle 129.1^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$  ;  $\text{Im}(y_S) = -1.855$  ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$  ;  $\text{Im}(y_S) = 1.855$  ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 177.5^\circ$  ;  $\text{Im}(y_L) = -1.352$  ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 53.4^\circ$  ;  $\text{Im}(y_L) = 1.352$  ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.471 = 5.404\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.404\text{dB} + 1.634\text{dB} = 9.734\text{dB}$

d)  $P_{\text{in}} = 140\mu\text{W} = -8.539\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -8.539\text{dBm} + 9.734\text{dB} = 1.195\text{dBm} = 1.317\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$  ;  $|S_{22}| = 0.560 < 1$  ;  $K = 1.091 > 1$  ;  $|\Delta| = |(0.037) + j \cdot (0.370)| = 0.371 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 24**

1.  $Z_L = 40\Omega$  parallel with  $1.19\text{nH}$  inductor at  $6.9\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0250\text{S} + j \cdot (-0.0194)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 1.250 + j \cdot (-0.969)$  then b)  $z = 1/y = 0.500 + j \cdot (0.387)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.765 + j \cdot 0.710 - 1) / (0.765 + j \cdot 0.710 + 1)$

$\Gamma = (0.025) + j \cdot (0.392) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.393 \angle 86.4^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $46\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.4\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 47.958\Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.1\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.112 \cdot \pi = 0.351$ ;  $\tan(\beta \cdot l) = 0.366$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 46.44\Omega + j \cdot (1.25)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 27.961\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 9) = 2\pi / 9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.549\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.668 \angle -179.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.557 \angle 133.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 155.5^\circ$ ;  $\text{Im}(y_S) = -1.795$ ;  $\theta_{p1} = 119.1^\circ$  or  $\theta_{S2} = 23.5^\circ$ ;  $\text{Im}(y_S) = 1.795$ ;  $\theta_{p2} = 60.9^\circ$

output:  $\theta_{L1} = 175.4^\circ$ ;  $\text{Im}(y_L) = -1.341$ ;  $\theta_{p1} = 126.7^\circ$  or  $\theta_{L2} = 51.6^\circ$ ;  $\text{Im}(y_L) = 1.341$ ;  $\theta_{p2} = 53.3^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.806 = 2.567\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.450 = 1.613\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.342 = 5.240\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.567\text{dB} + 5.240\text{dB} + 1.613\text{dB} = 9.419\text{dB}$

d)  $P_{in} = 85\mu\text{W} = -10.706\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -10.706\text{dBm} + 9.419\text{dB} = -1.287\text{dBm} = 0.744\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.668 < 1$ ;  $|S_{22}| = 0.557 < 1$ ;  $K = 1.110 > 1$ ;  $|\Delta| = |(0.107) + j \cdot (0.332)| = 0.349 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 25**

1.  $Z_L = 61\Omega$  series with  $0.25\text{pF}$  capacitor at  $9.2\text{GHz}$ . It's easier to compute first: b)  $Z_L = 61.00\Omega + j \cdot (-69.20)\Omega$ ,  $z = Z_L / 50\Omega = 1.220 + j \cdot (-1.384)$  then a)  $y = 1/z = 0.358 + j \cdot (0.407)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.075 - j \cdot 0.760 - 1) / (1.075 - j \cdot 0.760 + 1)$

$\Gamma = (0.150) + j \cdot (-0.311) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.346 \angle -64.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $49\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 49.497\Omega$  and the line physical length is  $l_1 = \lambda_1 / 4 = c / 4 / f_1$ .

At  $f_2 = 4.3\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c / f_2) \cdot (c / 4 / f_1) = \pi / 2 \cdot f_2 / f_1 = 0.239 \cdot \pi = 0.750$ ;  $\tan(\beta \cdot l) = 0.933$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 49.46\Omega + j \cdot (0.50)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 25.000\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 8) = 2\pi / 8$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.663\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.696 \angle 166.4^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 117.4^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 163.9^\circ$ ;  $\text{Im}(y_S) = -1.939$ ;  $\theta_{p1} = 117.3^\circ$  or  $\theta_{S2} = 29.7^\circ$ ;  $\text{Im}(y_S) = 1.939$ ;  $\theta_{p2} = 62.7^\circ$

output:  $\theta_{L1} = 3.3^\circ$ ;  $\text{Im}(y_L) = -1.352$ ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 59.3^\circ$ ;  $\text{Im}(y_L) = 1.352$ ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.940 = 2.877\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.897 = 5.907\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.877\text{dB} + 5.907\text{dB} + 1.634\text{dB} = 10.418\text{dB}$

d)  $P_{in} = 80\mu\text{W} = -10.969\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -10.969\text{dBm} + 10.418\text{dB} = -0.551\text{dBm} = 0.881\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.696 < 1$ ;  $|S_{22}| = 0.560 < 1$ ;  $K = 1.046 > 1$ ;  $|\Delta| = |(-0.063) + j \cdot (0.353)| = 0.358 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 26**

1.  $Z_L = 44\Omega$  series with 1.24nH inductor at 8.4GHz . It's easier to compute first: b)  $Z_L = 44.00\Omega + j \cdot (65.45)\Omega$ ,  $z = Z_L/50\Omega = 0.880 + j \cdot (1.309)$  then a)  $y = 1/z = 0.354 + j \cdot (-0.526)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.175 - j \cdot 0.875 - 1) / (1.175 - j \cdot 0.875 + 1)$

$\Gamma = (0.209) + j \cdot (-0.318) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.381 \angle -56.8^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $48\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.8\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 48.990 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 3.1\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.199 \cdot \pi = 0.624$ ;  $\tan(\beta \cdot l) = 0.720$

$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 48.67\Omega + j \cdot (0.94)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 24.900\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/9) = 2\pi/9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.397\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -150.2^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 146.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 140.0^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 10.2^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 168.7^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 45.3^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.000 = 4.771\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.771\text{dB} + 1.565\text{dB} = 8.624\text{dB}$

d)  $P_{in} = 120\mu\text{W} = -9.208\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -9.208\text{dBm} + 8.624\text{dB} = -0.584\text{dBm} = 0.874\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.182 > 1$ ;  $|\Delta| = |(0.246) + j \cdot (0.141)| = 0.284 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 27**

1.  $Z_L = 72\Omega$  parallel with 0.83nH inductor at 8.4GHz . It's easier to compute first: a)  $Y_L = 0.0139\text{S} + j \cdot (-0.0228)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 0.694 + j \cdot (-1.141)$  then b)  $z = 1/y = 0.389 + j \cdot (0.639)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.745 + j \cdot 0.985 - 1) / (0.745 + j \cdot 0.985 + 1)$

$\Gamma = (0.131) + j \cdot (0.491) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.508 \angle 75.1^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $74\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.8\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 60.828 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 3.8\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.194 \cdot \pi = 0.609$ ;  $\tan(\beta \cdot l) = 0.698$

$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 63.95\Omega + j \cdot (-11.84)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 18.132\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/12) = 2\pi/12$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.618\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.668 \angle -108.2^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.278 \angle -160.2^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 120.1^\circ$ ;  $\text{Im}(y_S) = -1.795$ ;  $\theta_{p1} = 119.1^\circ$  or  $\theta_{S2} = 168.1^\circ$ ;  $\text{Im}(y_S) = 1.795$ ;  $\theta_{p2} = 60.9^\circ$



output:  $\theta_{L1} = 133.2^\circ$  ;  $\text{Im}(y_L) = -0.579$  ;  $\theta_{p1} = 149.9^\circ$  or  $\theta_{L2} = 27.0^\circ$  ;  $\text{Im}(y_L) = 0.579$  ;  $\theta_{p2} = 30.1^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.806 = 2.567\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.084 = 0.349\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.049 = 4.841\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.567\text{dB} + 4.841\text{dB} + 0.349\text{dB} = 7.757\text{dB}$

d)  $P_{\text{in}} = 100\mu\text{W} = -10.000\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -10.000\text{dBm} + 7.757\text{dB} = -2.243\text{dBm} = 0.597\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.668 < 1$  ;  $|S_{22}| = 0.278 < 1$  ;  $K = 1.180 > 1$  ;  $|\Delta| = |(-0.228) + j \cdot (-0.125)| = 0.260 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 28**

1.  $Z_L = 55\Omega$  paralel with  $1.62\text{nH}$  inductor at  $7.0\text{GHz}$  . It's easier to compute first: a)  $Y_L = 0.0182\text{S} + j \cdot (-0.0140)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 0.909 + j \cdot (-0.702)$  then b)  $z = 1/y = 0.689 + j \cdot (0.532)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.860 + j \cdot 1.220 - 1) / (0.860 + j \cdot 1.220 + 1)$

$\Gamma = (0.248) + j \cdot (0.493) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.552 \angle 63.3^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $47\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.2\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 48.477 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 3.1\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.189 \cdot \pi = 0.594$ ;  $\tan(\beta \cdot l) = 0.675$

$Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 47.90\Omega + j \cdot (1.37)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 19.211\Omega$ .

For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$  ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/8) = 2\pi/8$  ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 1.076\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$  ;  $\Gamma_S = 0.700 \angle -101.0^\circ$  ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$  ;  $\Gamma_L = 0.310 \angle -149.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 117.7^\circ$  ;  $\text{Im}(y_S) = -1.960$  ;  $\theta_{p1} = 117.0^\circ$  or  $\theta_{S2} = 163.3^\circ$  ;  $\text{Im}(y_S) = 1.960$  ;  $\theta_{p2} = 63.0^\circ$

output:  $\theta_{L1} = 128.5^\circ$  ;  $\text{Im}(y_L) = -0.652$  ;  $\theta_{p1} = 146.9^\circ$  or  $\theta_{L2} = 20.5^\circ$  ;  $\text{Im}(y_L) = 0.652$  ;  $\theta_{p2} = 33.1^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.961 = 2.924\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.106 = 0.439\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.723 = 4.350\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.924\text{dB} + 4.350\text{dB} + 0.439\text{dB} = 7.713\text{dB}$

d)  $P_{\text{in}} = 85\mu\text{W} = -10.706\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -10.706\text{dBm} + 7.713\text{dB} = -2.993\text{dBm} = 0.502\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.700 < 1$  ;  $|S_{22}| = 0.310 < 1$  ;  $K = 1.083 > 1$  ;  $|\Delta| = |(-0.278) + j \cdot (-0.095)| = 0.294 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 29**

1.  $Z_L = 32\Omega$  paralel with  $1.31\text{nH}$  inductor at  $6.5\text{GHz}$  . It's easier to compute first: a)  $Y_L = 0.0313\text{S} + j \cdot (-0.0187)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 1.563 + j \cdot (-0.935)$  then b)  $z = 1/y = 0.471 + j \cdot (0.282)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.020 - j \cdot 0.765 - 1) / (1.020 - j \cdot 0.765 + 1)$

$\Gamma = (0.134) + j \cdot (-0.328) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.354 \angle -67.8^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $59\Omega$  load to a  $50\Omega$  source at  $f_1 = 6.6\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 54.314 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 4.0\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.303 \cdot \pi = 0.952$ ;  $\tan(\beta \cdot l) = 1.404$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 52.71\Omega + j \cdot (-4.13)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 21.635\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 7) = 2\pi / 7$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.913\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.644 \angle -161.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.551 \angle 139.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 145.5^\circ$ ;  $\text{Im}(y_S) = -1.684$ ;  $\theta_{p1} = 120.7^\circ$  or  $\theta_{S2} = 15.5^\circ$ ;  $\text{Im}(y_S) = 1.684$ ;  $\theta_{p2} = 59.3^\circ$

output:  $\theta_{L1} = 172.2^\circ$ ;  $\text{Im}(y_L) = -1.321$ ;  $\theta_{p1} = 127.1^\circ$  or  $\theta_{L2} = 48.8^\circ$ ;  $\text{Im}(y_L) = 1.321$ ;  $\theta_{p2} = 52.9^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.709 = 2.326\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.436 = 1.571\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.151 = 4.984\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.326\text{dB} + 4.984\text{dB} + 1.571\text{dB} = 8.882\text{dB}$

d)  $P_{in} = 100\mu\text{W} = -10.000\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -10.000\text{dBm} + 8.882\text{dB} = -1.118\text{dBm} = 0.773\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.644 < 1$ ;  $|S_{22}| = 0.551 < 1$ ;  $K = 1.162 > 1$ ;  $|\Delta| = |(0.198) + j \cdot (0.225)| = 0.299 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 30**

1.  $Z_L = 52\Omega$  series with  $0.48\text{pF}$  capacitor at  $7.4\text{GHz}$ . It's easier to compute first: b)  $Z_L = 52.00\Omega + j \cdot (-44.81)\Omega$ ,  $z = Z_L / 50\Omega = 1.040 + j \cdot (-0.896)$  then a)  $y = 1/z = 0.552 + j \cdot (0.475)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.900 - j \cdot 0.900 - 1) / (0.900 - j \cdot 0.900 + 1)$

$\Gamma = (0.140) + j \cdot (-0.407) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.431 \angle -71.0^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $36\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.3\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 42.426\Omega$  and the line physical length is  $l_1 = \lambda_1 / 4 = c / 4 / f_1$ .

At  $f_2 = 2.8\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c / f_2) \cdot (c / 4 / f_1) = \pi / 2 \cdot f_2 / f_1 = 0.151 \cdot \pi = 0.473$ ;  $\tan(\beta \cdot l) = 0.512$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 38.22\Omega + j \cdot (5.11)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 18.744\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 13) = 2\pi / 13$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.365\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.660 \angle -110.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.270 \angle -163.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 120.6^\circ$ ;  $\text{Im}(y_S) = -1.757$ ;  $\theta_{p1} = 119.6^\circ$  or  $\theta_{S2} = 169.4^\circ$ ;  $\text{Im}(y_S) = 1.757$ ;  $\theta_{p2} = 60.4^\circ$

output:  $\theta_{L1} = 134.3^\circ$ ;  $\text{Im}(y_L) = -0.561$ ;  $\theta_{p1} = 150.7^\circ$  or  $\theta_{L2} = 28.7^\circ$ ;  $\text{Im}(y_L) = 0.561$ ;  $\theta_{p2} = 29.3^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.772 = 2.484\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.079 = 0.329\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.133 = 4.959\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.484\text{dB} + 4.959\text{dB} + 0.329\text{dB} = 7.772\text{dB}$

d)  $P_{in} = 85\mu\text{W} = -10.706\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -10.706\text{dBm} + 7.772\text{dB} = -2.933\text{dBm} = 0.509\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.660 < 1$ ;  $|S_{22}| = 0.270 < 1$ ;  $K = 1.206 > 1$ ;  $|\Delta| = |(-0.216) + j \cdot (-0.130)| = 0.252 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 31**

1.  $Z_L = 36\Omega$  paralel with 1.16nH inductor at 9.9GHz . It's easier to compute first: a)  $Y_L = 0.0278S + j \cdot (-0.0139)S$ ,  $y = Y_L \cdot 50\Omega = 1.389 + j \cdot (-0.693)$  then b)  $z = 1/y = 0.576 + j \cdot (0.288)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.105 + j \cdot 0.765 - 1) / (1.105 + j \cdot 0.765 + 1)$

$\Gamma = (0.161) + j \cdot (0.305) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.345 \angle 62.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $72\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.7\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 60.000 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.2\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.113 \cdot \pi = 0.356$ ;  $\tan(\beta \cdot l) = 0.372$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 68.34\Omega + j \cdot (-8.19)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 28.868\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/9) = 2\pi/9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.406\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle 172.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 123.7^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 0.2^\circ$ ;  $\text{Im}(y_L) = -1.352$ ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 56.1^\circ$ ;  $\text{Im}(y_L) = 1.352$ ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.663 = 5.639\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.639\text{dB} + 1.634\text{dB} = 9.968\text{dB}$

d)  $P_{in} = 145\mu\text{W} = -8.386\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -8.386\text{dBm} + 9.968\text{dB} = 1.582\text{dBm} = 1.440\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$ ;  $|S_{22}| = 0.560 < 1$ ;  $K = 1.103 > 1$ ;  $|\Delta| = |(0.007) + j \cdot (0.355)| = 0.355 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 32**

1.  $Z_L = 46\Omega$  paralel with 0.60pF capacitor at 6.7GHz . It's easier to compute first: a)  $Y_L = 0.0217S + j \cdot (0.0253)S$ ,  $y = Y_L \cdot 50\Omega = 1.087 + j \cdot (1.263)$  then b)  $z = 1/y = 0.391 + j \cdot (-0.455)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.835 + j \cdot 0.830 - 1) / (0.835 + j \cdot 0.830 + 1)$

$\Gamma = (0.095) + j \cdot (0.409) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.420 \angle 76.9^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $72\Omega$  load to a  $50\Omega$  source at  $f_1 = 10.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 60.000 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.1\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.105 \cdot \pi = 0.330$ ;  $\tan(\beta \cdot l) = 0.342$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 68.82\Omega + j \cdot (-7.73)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 32.596\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/7) = 2\pi/7$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.658\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -147.6^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 148.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 138.7^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 8.9^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 167.7^\circ$  ;  $\text{Im}(y_L) = -1.317$  ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 44.3^\circ$  ;  $\text{Im}(y_L) = 1.317$  ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.958 = 4.711\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.711\text{dB} + 1.565\text{dB} = 8.564\text{dB}$

d)  $P_{\text{in}} = 50\mu\text{W} = -13.010\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -13.010\text{dBm} + 8.564\text{dB} = -4.447\text{dBm} = 0.359\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$  ;  $|S_{22}| = 0.550 < 1$  ;  $K = 1.186 > 1$  ;  $|\Delta| = |(0.255) + j \cdot (0.119)| = 0.282 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 33**

1.  $Z_L = 25\Omega$  series with  $0.30\text{pF}$  capacitor at  $7.8\text{GHz}$ . It's easier to compute first: b)  $Z_L = 25.00\Omega + j \cdot (-68.01)\Omega$ ,  $z = Z_L/50\Omega = 0.500 + j \cdot (-1.360)$  then a)  $y = 1/z = 0.238 + j \cdot (0.648)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.290 - j \cdot 0.755 - 1) / (1.290 - j \cdot 0.755 + 1)$

$\Gamma = (0.212) + j \cdot (-0.260) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.335 \angle -50.7^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $60\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.0\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 54.772 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 2.0\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.143 \cdot \pi = 0.449$ ;  $\tan(\beta \cdot l) = 0.482$

$Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 57.82\Omega + j \cdot (-4.13)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 25.081\Omega$ .

For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$  ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/7) = 2\pi/7$  ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.534\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$  ;  $\Gamma_S = 0.672 \angle -107.3^\circ$  ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$  ;  $\Gamma_L = 0.282 \angle -158.8^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 119.8^\circ$  ;  $\text{Im}(y_S) = -1.815$  ;  $\theta_{p1} = 118.9^\circ$  or  $\theta_{S2} = 167.5^\circ$  ;  $\text{Im}(y_S) = 1.815$  ;  $\theta_{p2} = 61.1^\circ$

output:  $\theta_{L1} = 132.6^\circ$  ;  $\text{Im}(y_L) = -0.588$  ;  $\theta_{p1} = 149.6^\circ$  or  $\theta_{L2} = 26.2^\circ$  ;  $\text{Im}(y_L) = 0.588$  ;  $\theta_{p2} = 30.4^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.823 = 2.609\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.086 = 0.360\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.007 = 4.781\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.609\text{dB} + 4.781\text{dB} + 0.360\text{dB} = 7.750\text{dB}$

d)  $P_{\text{in}} = 55\mu\text{W} = -12.596\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -12.596\text{dBm} + 7.750\text{dB} = -4.847\text{dBm} = 0.328\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.672 < 1$  ;  $|S_{22}| = 0.282 < 1$  ;  $K = 1.167 > 1$  ;  $|\Delta| = |(-0.234) + j \cdot (-0.122)| = 0.264 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 34**

1.  $Z_L = 58\Omega$  series with  $1.75\text{nH}$  inductor at  $6.5\text{GHz}$ . It's easier to compute first: b)  $Z_L = 58.00\Omega + j \cdot (71.47)\Omega$ ,  $z = Z_L/50\Omega = 1.160 + j \cdot (1.429)$  then a)  $y = 1/z = 0.342 + j \cdot (-0.422)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.870 + j \cdot 0.705 - 1) / (0.870 + j \cdot 0.705 + 1)$

$\Gamma = (0.064) + j \cdot (0.353) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.359 \angle 79.8^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $64\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.2\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 56.569 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 2.9\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.177 \cdot \pi = 0.556$ ;  $\tan(\beta \cdot l) = 0.621$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 59.38\Omega + j \cdot (-6.58)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 40.311\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega \cdot C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda/6) = 2\pi/6$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.844\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.664 \angle -176.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.556 \angle 134.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 153.8^\circ$ ;  $\text{Im}(y_S) = -1.776$ ;  $\theta_{p1} = 119.4^\circ$  or  $\theta_{S2} = 22.2^\circ$ ;  $\text{Im}(y_S) = 1.776$ ;  $\theta_{p2} = 60.6^\circ$

output:  $\theta_{L1} = 174.9^\circ$ ;  $\text{Im}(y_L) = -1.338$ ;  $\theta_{p1} = 126.8^\circ$  or  $\theta_{L2} = 51.1^\circ$ ;  $\text{Im}(y_L) = 1.338$ ;  $\theta_{p2} = 53.2^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.789 = 2.525\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.447 = 1.606\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.309 = 5.197\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.525\text{dB} + 5.197\text{dB} + 1.606\text{dB} = 9.328\text{dB}$

d)  $P_{in} = 60\mu\text{W} = -12.218\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -12.218\text{dBm} + 9.328\text{dB} = -2.891\text{dBm} = 0.514\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.664 < 1$ ;  $|S_{22}| = 0.556 < 1$ ;  $K = 1.117 > 1$ ;  $|\Delta| = |(0.126) + j \cdot (0.316)| = 0.340 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 35**

1.  $Z_L = 66\Omega$  parallel with  $0.54\text{nH}$  inductor at  $7.8\text{GHz}$ . It's easier to compute first: a)  $Y_L = 0.0152\text{S} + j \cdot (-0.0378)\text{S}$ ,  $y = Y_L \cdot 50\Omega = 0.758 + j \cdot (-1.889)$  then b)  $z = 1/y = 0.183 + j \cdot (0.456)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.290 - j \cdot 0.765 - 1) / (1.290 - j \cdot 0.765 + 1)$

$\Gamma = (0.214) + j \cdot (-0.262) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.339 \angle -50.8^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $55\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.5\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 52.440\Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 4.1\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.241 \cdot \pi = 0.758$ ;  $\tan(\beta \cdot l) = 0.946$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 52.52\Omega + j \cdot (-2.50)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 16.903\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega \cdot C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda/14) = 2\pi/14$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.515\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.640 \angle -151.5^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 145.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 140.6^\circ$ ;  $\text{Im}(y_S) = -1.666$ ;  $\theta_{p1} = 121.0^\circ$  or  $\theta_{S2} = 10.9^\circ$ ;  $\text{Im}(y_S) = 1.666$ ;  $\theta_{p2} = 59.0^\circ$

output:  $\theta_{L1} = 169.2^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 45.8^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.694 = 2.289\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.017 = 4.796\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.289\text{dB} + 4.796\text{dB} + 1.565\text{dB} = 8.649\text{dB}$

d)  $P_{in} = 55\mu\text{W} = -12.596\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -12.596\text{dBm} + 8.649\text{dB} = -3.947\text{dBm} = 0.403\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.640 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.181 > 1$ ;  $|\Delta| = |(0.241) + j \cdot (0.152)| = 0.285 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 36**

1.  $Z_L = 42\Omega$  paralel with  $0.94nH$  inductor at  $7.1GHz$ . It's easier to compute first: a)  $Y_L = 0.0238S + j \cdot (-0.0238)S$ ,  $y = Y_L \cdot 50\Omega = 1.190 + j \cdot (-1.192)$  then b)  $z = 1/y = 0.419 + j \cdot (0.420)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.045 - j \cdot 1.195 - 1) / (1.045 - j \cdot 1.195 + 1)$

$\Gamma = (0.271) + j \cdot (-0.426) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.505 \angle -57.5^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $48\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.3GHz$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 48.990 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 2.4GHz$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.129 \cdot \pi = 0.405$ ;  $\tan(\beta \cdot l) = 0.429$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 48.30\Omega + j \cdot (0.71)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 30.000\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/10) = 2\pi/10$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.507pF$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.637 \angle -141.1^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 152.7^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 135.3^\circ$ ;  $\text{Im}(y_S) = -1.653$ ;  $\theta_{p1} = 121.2^\circ$  or  $\theta_{S2} = 5.8^\circ$ ;  $\text{Im}(y_S) = 1.653$ ;  $\theta_{p2} = 58.8^\circ$

output:  $\theta_{L1} = 165.3^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 42.0^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{max}} = 1 / (1 - |S_{11}|^2) = 1.683 = 2.260dB$ ; the gain from load match:  $G_{L_{max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565dB$ ;  $G_0 = |S_{21}|^2 = 2.863 = 4.568dB$ ;

The transducer power gain:  $G_T = G_{S_{max}} + G_0 + G_{L_{max}} = 2.260dB + 4.568dB + 1.565dB = 8.393dB$

d)  $P_{in} = 65\mu W = -11.871dBm$ ;  $P_{out} = P_{in} + G_T = -11.871dBm + 8.393dB = -3.478dBm = 0.449mW$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.637 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.201 > 1$ ;  $|\Delta| = |(0.265) + j \cdot (0.060)| = 0.272 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 37**

1.  $Z_L = 38\Omega$  paralel with  $0.26pF$  capacitor at  $9.7GHz$ . It's easier to compute first: a)  $Y_L = 0.0263S + j \cdot (0.0158)S$ ,  $y = Y_L \cdot 50\Omega = 1.316 + j \cdot (0.792)$  then b)  $z = 1/y = 0.558 + j \cdot (-0.336)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.935 + j \cdot 1.065 - 1) / (0.935 + j \cdot 1.065 + 1)$

$\Gamma = (0.207) + j \cdot (0.437) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.483 \angle 64.7^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $73\Omega$  load to a  $50\Omega$  source at  $f_1 = 9.1GHz$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 60.415 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4f_1$ .

At  $f_2 = 4.4GHz$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.242 \cdot \pi = 0.760$ ;  $\tan(\beta \cdot l) = 0.950$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 59.93\Omega + j \cdot (-11.39)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 41.138\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$ ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/9) = 2\pi/9$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.321pF$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.680 \angle 172.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.560 \angle 121.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 160.4^\circ$ ;  $\text{Im}(y_S) = -1.855$ ;  $\theta_{p1} = 118.3^\circ$  or  $\theta_{S2} = 27.6^\circ$ ;  $\text{Im}(y_S) = 1.855$ ;  $\theta_{p2} = 61.7^\circ$

output:  $\theta_{L1} = 1.5^\circ$  ;  $\text{Im}(y_L) = -1.352$  ;  $\theta_{p1} = 126.5^\circ$  or  $\theta_{L2} = 57.5^\circ$  ;  $\text{Im}(y_L) = 1.352$  ;  $\theta_{p2} = 53.5^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.860 = 2.695\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.457 = 1.634\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.764 = 5.756\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.695\text{dB} + 5.756\text{dB} + 1.634\text{dB} = 10.086\text{dB}$

d)  $P_{\text{in}} = 70\mu\text{W} = -11.549\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -11.549\text{dBm} + 10.086\text{dB} = -1.463\text{dBm} = 0.714\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.680 < 1$  ;  $|S_{22}| = 0.560 < 1$  ;  $K = 1.112 > 1$  ;  $|\Delta| = |(-0.006) + j \cdot (0.348)| = 0.348 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 38**

1.  $Z_L = 44\Omega$  series with  $0.48\text{nH}$  inductor at  $8.8\text{GHz}$ . It's easier to compute first: b)  $Z_L = 44.00\Omega + j \cdot (26.54)\Omega$ ,  $z = Z_L/50\Omega = 0.880 + j \cdot (0.531)$  then a)  $y = 1/z = 0.833 + j \cdot (-0.503)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.885 - j \cdot 0.875 - 1) / (0.885 - j \cdot 0.875 + 1)$

$\Gamma = (0.127) + j \cdot (-0.405) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.425 \angle -72.6^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $57\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.5\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 53.385 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 2.2\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.147 \cdot \pi = 0.461$ ;  $\tan(\beta \cdot l) = 0.496$

$Z_{\text{in}} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 55.46\Omega + j \cdot (-2.90)\Omega$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 28.636\Omega$ .

For a section of an open-circuited transmission line  $Z_{\text{in}} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1/\omega/C$  ;  $\beta \cdot l = 2\pi/\lambda \cdot (\lambda/8) = 2\pi/8$  ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.695\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$  ;  $\Gamma_S = 0.664 \angle -109.1^\circ$  ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$  ;  $\Gamma_L = 0.274 \angle -161.6^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 120.4^\circ$  ;  $\text{Im}(y_S) = -1.776$  ;  $\theta_{p1} = 119.4^\circ$  or  $\theta_{S2} = 168.7^\circ$  ;  $\text{Im}(y_S) = 1.776$  ;  $\theta_{p2} = 60.6^\circ$

output:  $\theta_{L1} = 133.8^\circ$  ;  $\text{Im}(y_L) = -0.570$  ;  $\theta_{p1} = 150.3^\circ$  or  $\theta_{L2} = 27.8^\circ$  ;  $\text{Im}(y_L) = 0.570$  ;  $\theta_{p2} = 29.7^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.789 = 2.525\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.081 = 0.339\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.091 = 4.900\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.525\text{dB} + 4.900\text{dB} + 0.339\text{dB} = 7.764\text{dB}$

d)  $P_{\text{in}} = 110\mu\text{W} = -9.586\text{dBm}$ ;  $P_{\text{out}} = P_{\text{in}} + G_T = -9.586\text{dBm} + 7.764\text{dB} = -1.822\text{dBm} = 0.657\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.664 < 1$  ;  $|S_{22}| = 0.274 < 1$  ;  $K = 1.193 > 1$  ;  $|\Delta| = |(-0.222) + j \cdot (-0.128)| = 0.256 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

### **Subject no. 39**

1.  $Z_L = 72\Omega$  series with  $0.62\text{pF}$  capacitor at  $9.0\text{GHz}$ . It's easier to compute first: b)  $Z_L = 72.00\Omega + j \cdot (-28.52)\Omega$ ,  $z = Z_L/50\Omega = 1.440 + j \cdot (-0.570)$  then a)  $y = 1/z = 0.600 + j \cdot (0.238)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (1.060 + j \cdot 0.940 - 1) / (1.060 + j \cdot 0.940 + 1)$

$\Gamma = (0.196) + j \cdot (0.367) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.416 \angle 61.8^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $56\Omega$  load to a  $50\Omega$  source at  $f_1 = 7.4\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 52.915 \Omega$  and the line physical length is  $l_1 = \lambda_1/4 = c/4/f_1$ .

At  $f_2 = 3.0\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c/f_2) \cdot (c/4/f_1) = \pi/2 \cdot f_2/f_1 = 0.203 \cdot \pi = 0.637$ ;  $\tan(\beta \cdot l) = 0.740$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 53.72\Omega + j \cdot (-2.91)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 28.868\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 11) = 2\pi / 11$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.417\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.652 \angle -167.0^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.553 \angle 137.0^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 148.8^\circ$ ;  $\text{Im}(y_S) = -1.720$ ;  $\theta_{p1} = 120.2^\circ$  or  $\theta_{S2} = 18.2^\circ$ ;  $\text{Im}(y_S) = 1.720$ ;  $\theta_{p2} = 59.8^\circ$

output:  $\theta_{L1} = 173.3^\circ$ ;  $\text{Im}(y_L) = -1.327$ ;  $\theta_{p1} = 127.0^\circ$  or  $\theta_{L2} = 49.7^\circ$ ;  $\text{Im}(y_L) = 1.327$ ;  $\theta_{p2} = 53.0^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.739 = 2.404\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.441 = 1.585\text{dB}$ ;  $G_0 = |S_{21}|^2 = 3.211 = 5.067\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.404\text{dB} + 5.067\text{dB} + 1.585\text{dB} = 9.056\text{dB}$

d)  $P_{in} = 105\mu\text{W} = -9.788\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -9.788\text{dBm} + 9.056\text{dB} = -0.732\text{dBm} = 0.845\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.652 < 1$ ;  $|S_{22}| = 0.553 < 1$ ;  $K = 1.143 > 1$ ;  $|\Delta| = |(0.174) + j \cdot (0.263)| = 0.316 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)

#### **Subject no. 40**

1.  $Z_L = 26\Omega$  series with  $0.67\text{nH}$  inductor at  $9.8\text{GHz}$ . It's easier to compute first: b)  $Z_L = 26.00\Omega + j \cdot (41.26)\Omega$ ,  $z = Z_L / 50\Omega = 0.520 + j \cdot (0.825)$  then a)  $y = 1/z = 0.547 + j \cdot (-0.867)$

2. a)  $\Gamma = (z - 1) / (z + 1) = (0.900 - j \cdot 0.990 - 1) / (0.900 - j \cdot 0.990 + 1)$

$\Gamma = (0.172) + j \cdot (-0.431) \leftrightarrow \text{Re}\Gamma + j \cdot \text{Im}\Gamma$  or  $\Gamma = 0.464 \angle -68.2^\circ \leftrightarrow |\Gamma| \angle \arg(\Gamma)$

3. The quarter wave transformer is designed to match a  $65\Omega$  load to a  $50\Omega$  source at  $f_1 = 8.2\text{GHz}$  so  $Z_1 = \sqrt{(Z_0 \cdot Z_L)} = 57.009\Omega$  and the line physical length is  $l_1 = \lambda_1 / 4 = c / 4 / f_1$ .

At  $f_2 = 3.0\text{GHz}$  the characteristic impedance and physical length do not change but  $\beta \cdot l = \beta_2 \cdot l_1 = 2 \cdot \pi / \lambda_2 \cdot l_1 = 2 \cdot \pi / (c / f_2) \cdot (c / 4 / f_1) = \pi / 2 \cdot f_2 / f_1 = 0.183 \cdot \pi = 0.575$ ;  $\tan(\beta \cdot l) = 0.648$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta l)} = 59.71\Omega + j \cdot (-7.17)\Omega$$

4.  $R = G = 0$ , thus the line is lossless,  $Z_0 = \sqrt{L/C} = 16.720\Omega$ .

For a section of an open-circuited transmission line  $Z_{in} = -j \cdot Z_0 \cdot \text{ctg}(\beta \cdot l) = 1 / (j \cdot \omega \cdot C) = -j \cdot 1 / \omega / C$ ;  $\beta \cdot l = 2\pi / \lambda \cdot (\lambda / 12) = 2\pi / 12$ ;  $C = \text{tg}(\beta \cdot l) / (2 \cdot \pi \cdot f) / Z_0 = 0.470\text{pF}$

5. a)  $S_{12} = 0$ , we have an unilateral transistor, maximum gain match of the source is when  $\Gamma_S = S_{11}^*$ ;  $\Gamma_S = 0.638 \angle -142.4^\circ$ ; Maximum gain match of the load  $\Gamma_L = S_{22}^*$ ;  $\Gamma_L = 0.550 \angle 151.8^\circ$

Formulae from L6/2019, S97-101, two solutions for the input match, two solutions for the output match, all lines have  $Z_0 = 50\Omega$

input:  $\theta_{S1} = 136.0^\circ$ ;  $\text{Im}(y_S) = -1.657$ ;  $\theta_{p1} = 121.1^\circ$  or  $\theta_{S2} = 6.4^\circ$ ;  $\text{Im}(y_S) = 1.657$ ;  $\theta_{p2} = 58.9^\circ$

output:  $\theta_{L1} = 165.8^\circ$ ;  $\text{Im}(y_L) = -1.317$ ;  $\theta_{p1} = 127.2^\circ$  or  $\theta_{L2} = 42.4^\circ$ ;  $\text{Im}(y_L) = 1.317$ ;  $\theta_{p2} = 52.8^\circ$

b) The shunt stubs **must** be placed in parallel with the  $50\Omega$  source/load

c) The gain from source match:  $G_{S_{\max}} = 1 / (1 - |S_{11}|^2) = 1.686 = 2.270\text{dB}$ ; the gain from load match:  $G_{L_{\max}} = 1 / (1 - |S_{22}|^2) = 1.434 = 1.565\text{dB}$ ;  $G_0 = |S_{21}|^2 = 2.883 = 4.599\text{dB}$ ;

The transducer power gain:  $G_T = G_{S_{\max}} + G_0 + G_{L_{\max}} = 2.270\text{dB} + 4.599\text{dB} + 1.565\text{dB} = 8.433\text{dB}$

d)  $P_{in} = 105\mu\text{W} = -9.788\text{dBm}$ ;  $P_{out} = P_{in} + G_T = -9.788\text{dBm} + 8.433\text{dB} = -1.355\text{dBm} = 0.732\text{mW}$

e) Stability can be analyzed in different ways, in this case the most simple one:  $|S_{11}| = 0.638 < 1$ ;  $|S_{22}| = 0.550 < 1$ ;  $K = 1.197 > 1$ ;  $|\Delta| = |(0.265) + j \cdot (0.072)| = 0.274 < 1$ , thus the transistor is unconditionally stable, in particular is stable including the match designed at a)