PROBLEMS SOLVED

Problem 1

An antenna has a beam solid angle that is equivalent to a trpezoidal patch on the surface of a sphere of radius r. The angular space of the patch on the surface of the sphere extends between $\pi/6 \le \theta \le \pi$ in latitude and $\pi/4 \le \phi \le \pi/3$ in longitude.

a) Find, exatly, the equivalent beam solid angle.

b) Corresponding antenna maximum directivity in dB.

Solution

$$\text{a) } \Omega = \int_{\phi=\pi/4}^{\phi=\pi/3} \int_{\theta=\pi/6}^{\pi} sin\theta d\theta d\phi = \int_{\phi=\pi/4}^{\phi=\pi/3} d\phi \int_{\theta=\pi/6}^{\pi} sin\theta d\theta = \left(\frac{\pi}{3} - \frac{\pi}{4}\right) \int_{\theta=\pi/6}^{\pi} sin\theta d\theta =$$

$$= \frac{\pi}{12}\left[\left(-cos(\pi)\right) - \left(-cos\left(\frac{\pi}{6}\right)\right)\right] = \frac{\pi}{12} \cdot \left[1 + \frac{\sqrt{3}}{2}\right] = 0448 \text{ rad.}$$

$$\text{b) } D_0 = \frac{4\pi}{\Omega} = \frac{4\pi}{0.448} 28.05 = 14.48 dB$$

Problem 2

A hypotetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field $\left(E_\theta\right)$ is measured to be 5 V/m. Find the

a) power radiated $\left(P_{rad}\right)$

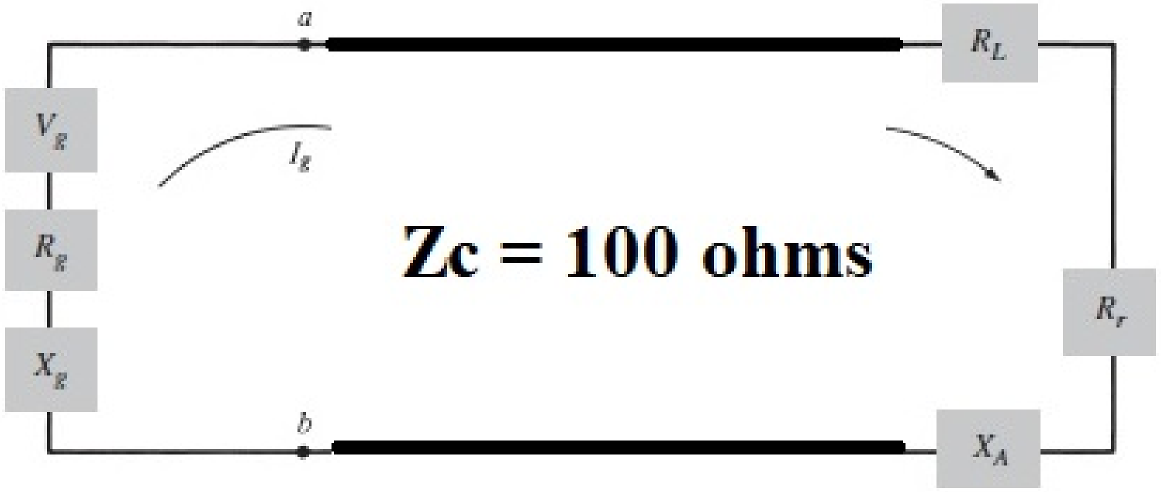
b) power density $\left(S\right)$.

Solution

a) $P_{rad} = \frac{1}{2}\frac{E_\theta^2}{\eta_0}4\pi r^2 = \frac{1}{2}\frac{5^2}{120\pi}4\pi 10^4 \text{ W} = \frac{5}{12}10^4\, W = 4.16\, kW$

b) $S = \frac{P_{rad}}{4\pi r^2} = \frac{1}{2}\frac{E_\theta^2}{\eta_0} = \frac{5^2}{2\cdot 120\pi}\frac{W}{m^2} = 0.033\frac{W}{m^2}$

Problem 3



An antenna with a radiation resistance of 48 ohms, a loss resistance of 2 ohms, and a reactance of 50 ohms is connected to a generator with open-circuit of 10 V and internal impedance of 50 ohms via a $\lambda/4-$long transmission line with characteristic impedance of 100 ohms. Determine the power radiated by the antenna.

Solution

$V_g = 10V, R_g = 50\Omega, X_g = 0$

$R_r = 48\Omega, R_L = 2\Omega, X_A = 50\Omega$

Input impedance in a transmission line, seen from generator, is given by:

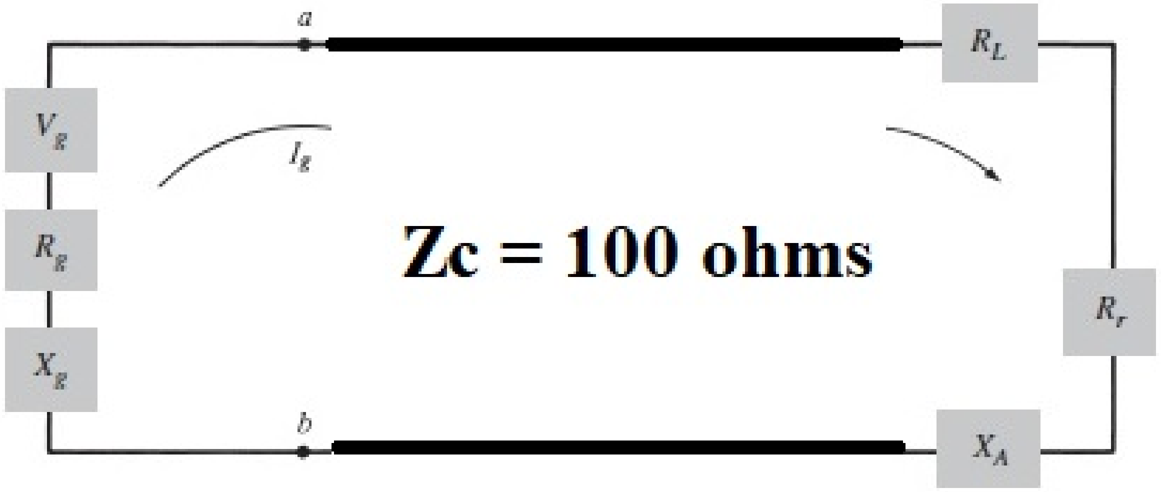
$$\mathrm{Z}_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

For us: $\mathrm{Z}_0 =$ characteristic impedance = 100Ω, l = $\lambda/4$. So:

$$\mathrm{Z}_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R_r + R_L + jX_A}$$

The current, I_g, is:

Problem 3 cont.



$V_g = 10V, R_g = 50\Omega, X_g = 0$

$R_r = 48\Omega, R_L = 2\Omega, X_A = 50\Omega$

The current, I_g, is:

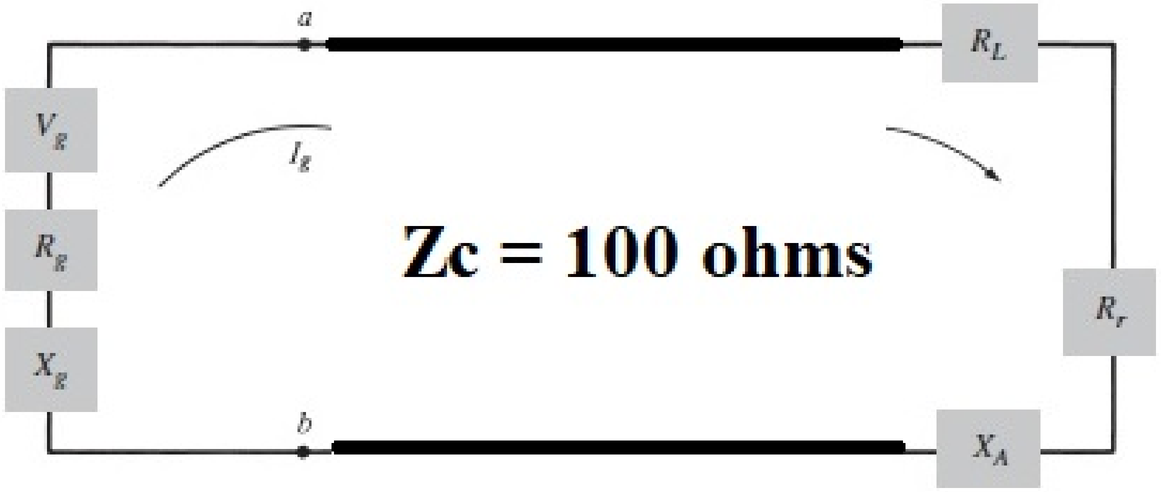
$$I_g = \frac{V_g}{Z_g + Z_{in}} = \frac{V_g}{R_g + R_{in} + jX_{in}}; \quad |I_g| = \frac{|V_g|}{\sqrt{(R_g + R_{in})^2 + X_{in}^2}}$$

The radiated power = $P_r = \frac{1}{2}|I_g|^2 R_r$

$$R_{in} = \frac{Z_0^2 (R_r + R_L)}{(R_r + R_L)^2 + X_A^2} = \frac{100^2 (48+2)}{(48+2)^2 + 50^2} = \frac{100^2 \cdot 50}{2 \cdot 50^2} = 100\Omega$$

$$X_{in} = -\frac{Z_0^2 X_A}{(R_r + R_L)^2 + X_A^2} = -\frac{100^2 \cdot 50}{(48+2)^2 + 50^2} = -100\Omega$$

Problem 3 cont.



$$P_r = \frac{1}{2}\left|I_g\right|^2 R_r = \frac{1}{2}\frac{\left|V_g\right|^2 R_r}{\left(R_g + R_{in}\right)^2 + X_{in}^2} = \frac{1}{2}\frac{10^2 \cdot 48}{\left(50+100\right)^2 + 100^2} = \frac{24}{15^2+10^2} = \frac{24}{225+100} = \frac{24}{325} = 0.0738W$$

WORNING

At the exam will be either problems of the type in this slide, or problems like the ones in the course examples.

La examen se vor da fie probleme de tipul celor din acest slide , fie probleme de tipul celor din exemplele din curs.